



Digital version

CS105 Ordinary Differential Equations Section B - Homework 1

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Problem 1.

Find the general solutions of the ODEs:

a) $\frac{dy}{dt} = 5y - 4$

$$\frac{dy}{dt} = 5y - 4 \xrightarrow{y \neq 0.8} \frac{y'}{y - 0.8} = 5 \implies \int \frac{y'}{y - 0.8} dt = \int 5 dt \implies \ln |y - 0.8| = 5t + c_1$$

$$\therefore y = 0.8 \text{ or } y \neq 0.8 \implies y = 0.8 + ce^{5t}$$

b) $\frac{-1}{3} \frac{dz}{dy} + y^2 = 2z$

$$\frac{dz}{dy} + 6z = 3y^2 \implies \mu(y) \frac{dz}{dy} + 6\mu(y)z = 3\mu(y)y^2, \text{ take } \frac{d\mu(y)}{dt} = 6\mu(y)$$

$$\mu(y) = 0 \text{ or } \mu(y) = ce^{6y} \implies ce^{6y} \frac{dz}{dy} + 6ce^{6y}z = (zce^{6y})' = 3ce^{6y}y^2$$

$$3 \int y^2 e^{6y} dy \stackrel{IbP}{=} \frac{y^2 e^{6y}}{2} - \frac{ye^{6y}}{6} + \frac{e^{6y}}{36} + c_1$$

$$zce^{6y} = 3c \int e^{6y} y^2 dy \implies z = \frac{y^2}{2} - \frac{y}{6} + \frac{1}{36} + \frac{c_1}{e^{6y}}$$

Problem 2.

Find the particular solutions of the ODEs in **Problem 1** with the corresponding initial conditions (IC):

a) $t = 0, y = 0$

$$0 = 0.8 + ce^0 \implies c = -0.8 \implies y = 0.8(1 - e^{5t})$$

b) $y = 1, z = \frac{2}{9}$

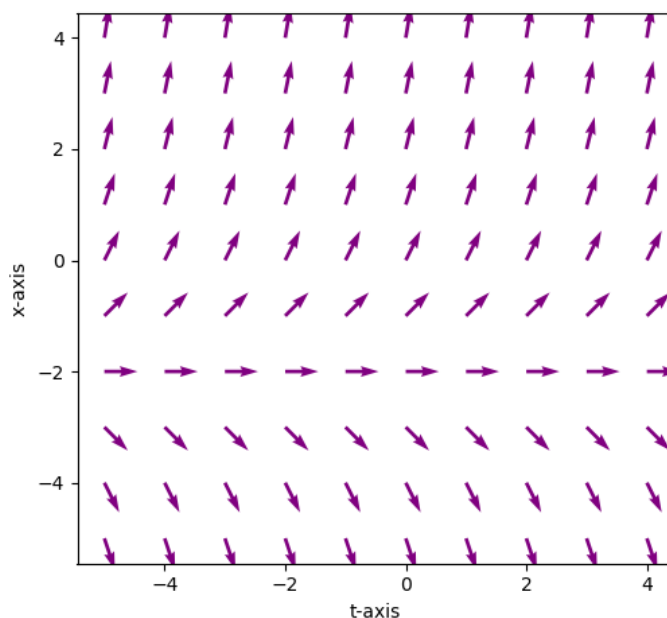
$$\frac{2}{9} = \frac{1}{2} - \frac{1}{6} + \frac{1}{36} + \frac{c_1}{e^6} \implies c_1 = -\frac{5e^6}{36} \implies z = \frac{y^2}{2} - \frac{y}{6} + \frac{1}{36} - \frac{5e^{6-6y}}{36}$$

Problems 3 and 4.

Draw the direction field of the following ODEs. You can draw by hand, but I encourage you to do this with programming. To get full points you need to draw at least 4 arrows in each quadrant.

1. Are there equilibrium solutions?
2. If there are equilibrium solutions, are they stable or unstable? Namely, do the other solutions seem to converge to them, or diverge away from them?
3. What is the asymptotic behavior of the solutions as $t \rightarrow \pm\infty$?

a) $\frac{dx}{dt} = x + 2$

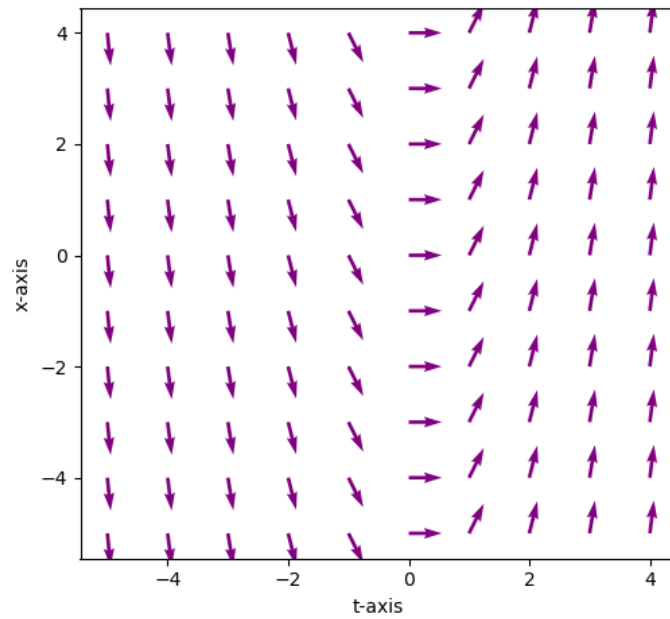


$$\frac{dx}{dt} = x + 2 \xrightarrow{x \neq -2} \frac{x'}{x + 2} = 1 \implies \ln |x + 2| = t + c_1$$

$$\therefore x = -2 \text{ or } x \neq -2 \implies x = ce^t - 2$$

As $t \rightarrow -\infty$, $x \rightarrow -2$, $t \rightarrow \infty$, $x \rightarrow \infty$ or $-\infty$, and at $x = -2$, x is independent of t . Therefore, $x = -2$ is an equilibrium solution, however it is an unstable equilibrium and even you we make a small change in x , it will diverge to infinity.

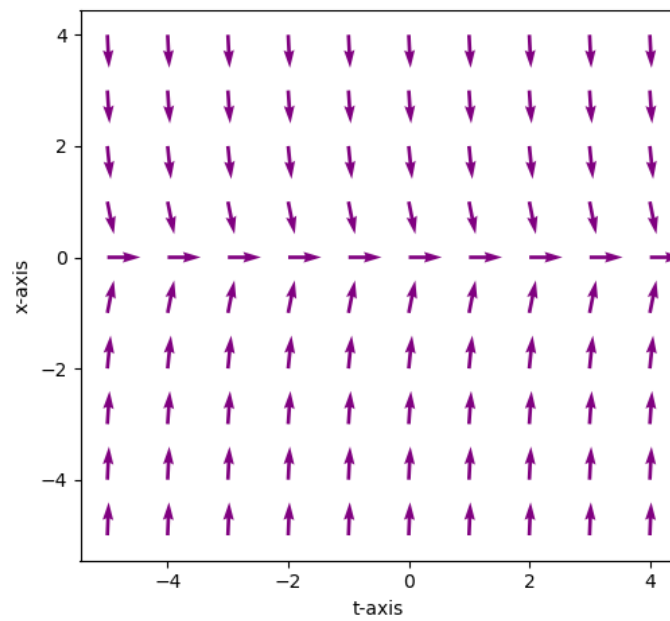
b) $\frac{dx}{dt} = 2t$



$$\frac{dx}{dt} = 2t \implies x = t^2 + c$$

We see multiple parabolas in the direction field. No equilibrium solutions and as $t \rightarrow \pm\infty$, $x \rightarrow \infty$.

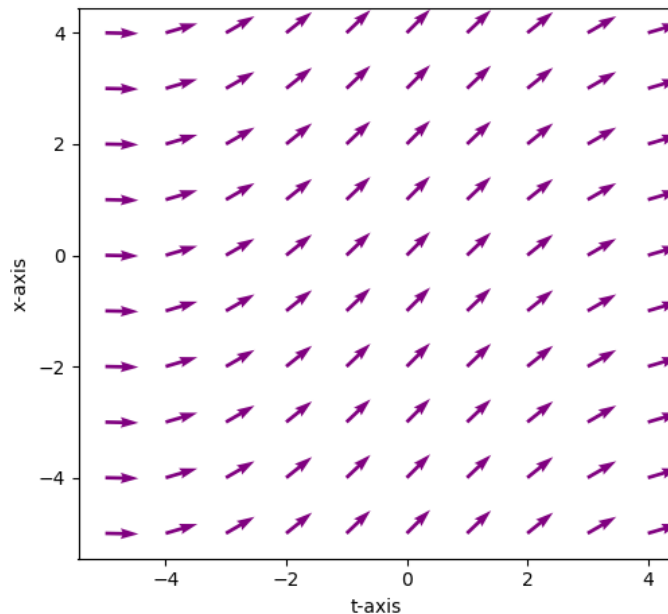
c) $\frac{dx}{dt} = -5x$



$$\frac{dx}{dt} = -5x \xrightarrow{x \neq 0} \frac{x'}{x} = -5 \implies x = ce^{-5t} \text{ or } x = 0$$

We see that $x = 0$ is a stable equilibrium solution and all other solutions get closer to it, as t increases. So, $t \rightarrow -\infty$, $x \rightarrow \infty$ or $-\infty$ and $t \rightarrow \infty$, $x \rightarrow 0$.

d) $\frac{dx}{dt} = \cos\left(\frac{t}{\pi}\right)$



$$\frac{dx}{dt} = \cos\left(\frac{t}{\pi}\right) \implies x = \frac{1}{\pi} \sin\left(\frac{t}{\pi}\right) + c$$

We see multiple sine waves in the direction field. No equilibrium solutions or asymptotic behavior.

The direction fields of these equations have a specific property - independence from one of the coordinate axes. Why is this? How would you change the ODEs to break this property?

We have two cases, either the solution has an added constant that shifts in the x direction, or a multiplied constant, that shifts in the t direction ($c_1 e^{at} \implies e^{c_2 + at}$, therefore this multiplied constant is actually an added constant for t).