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CS105 Ordinary Differential Equations - Homework 8

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All problems from Edwards and Penney. Page numbers are in the upper-right corner of each page.

Problem 19

Use the transforms in Fig. 4.1.2 to find the Laplace transforms of the functions in Problems 11 through 22. A preliminary in- tegration by parts may be necessary.

19) $f(t) = (1+t)^3$

$$f(t) = t^3 + 3t^2 + 3t + 1 \implies \mathcal{L}\left[f(t)\right](s) = \frac{1}{s} + \frac{3}{s^2} + \frac{6}{s^3} + \frac{6}{s^4}$$

Problem 30

Use the transforms in Fig. 4.1.2 to find the inverse Laplace transforms of the functions in Problems 23 through 32.

30) $F(s) = \dfrac{9+s}{4-s^2}$

$$\mathcal{L}^{-1}\left[F(s)\right] = \mathcal{L}^{-1}\left[-\frac{9}{2}\frac{2}{s^2-4} - \frac{s}{s^2-4}\right] = -4.5\sinh(2t) - \cosh(2t)$$

Problem 10

Use Laplace transforms to solve the initial value problems in Problems 1 through 16.

10) $x'' + 3x' + 2x = t$, $x(0) = 0$, $x'(0) = 2$.

$$\mathcal{L}\left[x'' + 3x' + 2x\right] = s^2F(s) - sx(0) - x'(0) + 3sF(s) - 3x(0) + 2F(s) = \frac{1}{s^2}$$

$$F(s) = \frac{1+2s^2}{s^2(s^2+3s+2)} = \frac{1+2s^2}{s^2(s+1)(s+2)} = -\frac{3}{4s} + \frac{1}{2s^2} + \frac{3}{s+1} - \frac{9}{4\,(s+2)}$$

$$\therefore x(t) = -\frac{3}{4} + \frac{t}{2} + 3e^{-t} - \frac{9}{4}e^{-2t}$$

Problem 1

Apply the translation theorem to find the Laplace transforms of the functions in Problems 1 through 4.

1) $f(t) = t^4 e^{\pi t}$

$$f(t) = t^4 e^{\pi t} \implies \mathcal{L}\left[f(t)\right] = \frac{4!}{(s-\pi)^5}$$

Problem 16

Use partial fractions to find the inverse Laplace transforms of the functions in Problems 11 through 22.

16) $F(s) = \frac{1}{(s^2+s-6)^2}$

$$\frac{1}{(s^2+s-6)^2} = \frac{1}{((x+3)(x-2))^2} = -\frac{2}{125\,(s-2)} + \frac{1}{25\,(s-2)^2} + \frac{2}{125\,(s+3)} + \frac{1}{25\,(s+3)^2}$$

$$\therefore f(t) = -\frac{2}{125}e^{2t} + \frac{1}{25}te^{2t} + \frac{2}{125}e^{-3t} + \frac{1}{25}te^{-3t}$$

Problem 6

Find the convolution $f(t) * g(t)$ in Problems 1 through 6

6) $f(t) = e^{at}$, $g(t) = e^{bt}$ $a \neq b$.

$$(f * g)(t) = \int_0^t e^{ax} e^{b(t-x)}\,\mathrm{d}x = e^{bt}\int_0^t e^{(a-b)x}\,\mathrm{d}x = \frac{e^{at}-e^{bt}}{a-b}$$

Problem 8

Apply the convolution theorem to find the inverse Laplace transforms of the functions in Problems 7 through 14.

8) $F(s) = \dfrac{1}{s(s^2+4)}$

$$F(s) = \frac{1}{(s)}\frac{1}{s^2+4}$$

$$(h * g) = \int_0^t \frac{1}{2}\sin(2x)\,\mathrm{d}x = \left[-\frac{1}{4}\cos(2x)\right]_0^t = \frac{1}{4} - \frac{1}{4}\cos(2t)$$