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# CS105 Ordinary Differential Equations - Homework 9

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## Part 1

### Page 307 Problem 14

$y'' - 4y' + 4y = 0$; $y(0) = 1$, $y'(0) = 1$

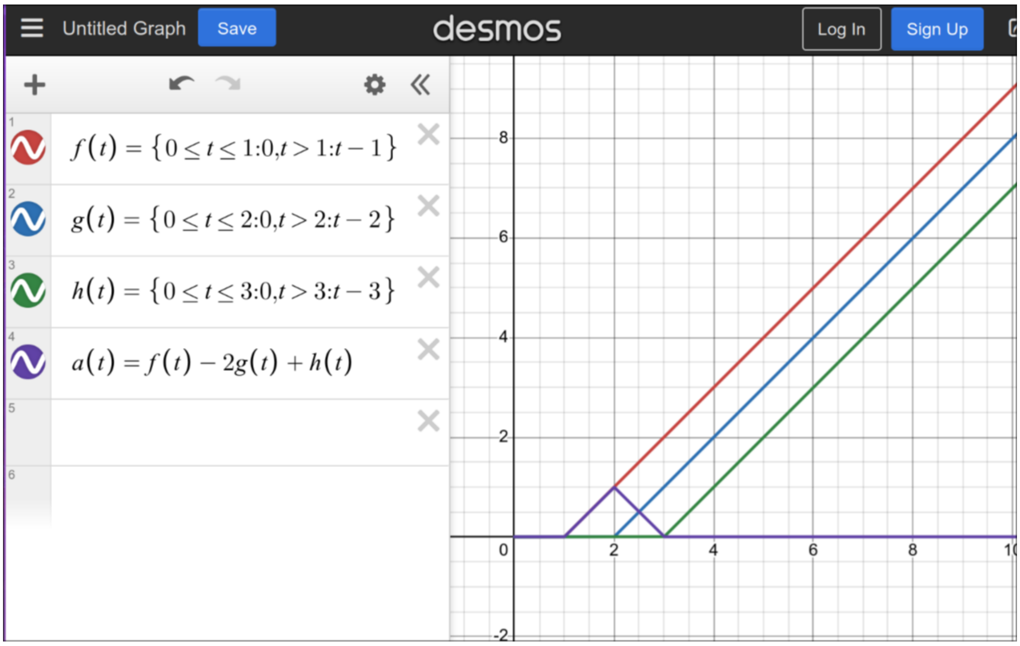
$$s^2Y(s) - sy(0) - y'(0) - 4(sY(s) - y(0)) + 4Y(s) = 0$$

$$s^2Y(s) - s - 1 - 4sY(s) + 4 + 4Y(s) = 0 \implies Y(s)(s^2 - 4s + 4) = s - 3$$

$$Y(s) = \frac{s-3}{(s-2)^2} = \frac{1}{s-2} - \frac{1}{(s-2)^2} \implies y(t) = e^{2t} - te^{2t}$$

### Page 314 Problem 6, then find its Laplace transform

In each of Problems 1 through 6 sketch the graph of the given function on the interval $t \geq 0$.
$f(t) = (t-1)u_1(t) - 2(t-2)u_2(t) + (t-3)u_3(t)$




$$\mathcal{L}[f(t)](s) = \frac{e^{-s}}{s^2} - \frac{2e^{-2s}}{s^2} + \frac{e^{-3s}}{s^2}$$

**Page 321 Problem 2**

In each of Problems 1 through 13 find the solution of the given initial value problem. Draw the graphs of the solution and of the forcing function; explain how they are related.

$y'' + 2y' + 2y = h(t)$; $y(0) = 0$, $y'(0) = 1$; $h(t) = \begin{cases} 1, \pi \leq t < 2\pi \\ 0, 0 \leq t < \pi \text{ and } t \geq 2\pi \end{cases}$

$$y'' + 2y' + 2y = u_\pi(t) - u_{2\pi}(t) - 1$$

$$Y(s)(s^2 + 2s + 2) = \frac{e^{-\pi s} - e^{-2\pi s}}{s} + 1$$

$$Y(s) = \frac{e^{-\pi s}}{s(s^2+2s+2)} - \frac{e^{-2\pi s}}{s(s^2+2s+2)} + \frac{1}{s^2+2s+2}$$

$$Y(s) = e^{-\pi s}\left(\frac{1}{2s} + \frac{\frac{-s-2}{2}}{(s+1)^2+1}\right) + e^{-2\pi s}\left(\frac{1}{2s} + \frac{\frac{-s-2}{2}}{(s+1)^2+1}\right) + \frac{1}{(s+1)^2+1}$$

$$\therefore y(t) = u_\pi(t)\left(\frac{1}{2} + \frac{e^{-(t-\pi)}\cos t}{2} + \frac{e^{-(t-\pi)}\sin t}{2}\right) - u_{2\pi}(t)\left(\frac{1}{2} - \frac{e^{-(t-2\pi)}\cos t}{2} - \frac{e^{-(t-2)}\sin t}{2}\right) + e^{-t}\sin t$$

**Page 328 Problem 5**

In each of Problems 1 through 12 find the solution of the given initial value problem and draw its graph.

$y'' + 2y' + 3y = \sin t + \delta(t - 3\pi)$; $y(0) = 0$, $y'(0) = 0$

$$s^2Y(s) + 2sY(s) + 3Y(s) = \frac{1}{s^2+1} + e^{-3\pi s}$$

$$Y(s) = \frac{1}{(s^2+2s+3)(s^2+1)} + \frac{e^{-3\pi s}}{s^2+2s+3}$$

$$Y(s) = \frac{0.25(s+1)}{(s+1)^2+2} - \frac{0.25(s-1)}{s^2+1} + \frac{e^{-3\pi s}}{(s+1)^2+2}$$

$$y(t) = \frac{1}{4}e^{-t}\cos\left(\sqrt{2}t\right) - \frac{1}{4}\cos(t) + \frac{1}{4}\sin(t) + \frac{1}{\sqrt{2}}u_{3\pi}e^{-(t-3\pi)}\sin\left(\sqrt(2)(t-3\pi)\right)$$

# Part 2

Suppose you have an LRC circuit that is described by the ODE.

$$LQ'' + RQ' + \frac{1}{C}Q = 0$$

You are too lazy to read the specs for the elements of the circuit and instead decide to use a clever trick to find the parameters L, R, C. You monitor the charge on the capacitor over time, $Q(t)$, after applying a short strong pulse of voltage at $t = 0$ to the system at rest. That is, $Q(0) = Q'(0) = 0$.

You tune your voltage pulse such that its time integral is equal to 1. As we discussed in class, the response $Q(t)$ to such an input should be almost the same as the response to a delta function $\delta(t)$. You find that the response $Q(t)$ closely follows the analytic function

$$Q(t) = \frac{1}{2}\left(e^{-t} - e^{-2t}\right)$$

Assuming this function is the actual response to $\delta(t)$, use it to find the parameters L, R, C.

$$LQ'' + RQ' + \frac{1}{C}Q = \delta(t)$$

$$LY(s)s^2 + RY(s)s + \frac{Y(s)}{C} = 1 \implies Y(s) = \frac{1}{Ls^2 + Rs + \frac{1}{C}}$$

$$Q(t) = \frac{1}{2}\left(e^{-t} - e^{-2t}\right) \implies Y(s) = \frac{\frac{1}{2}}{s+1} - \frac{\frac{1}{2}}{s+2} = \frac{1}{2s^2 + 6s + 4}$$

$$\therefore L = 2, R = 6, C = \frac{1}{4}$$