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ENGS104 Probability and Statistics - Homework 1

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Problem 1

Dave has two children. One child is a boy. What is the probability that the other child is a girl?

Assume the independent probability is 1/2, for being either a boy or a girl. We have the following probabilities:

- probability of both children being boys $P(BB) = \frac{1}{2} \cdot \frac{1}{2} = 1/4$;
- probability of both children being girls $P(GG) = \frac{1}{2} \cdot \frac{1}{2} = 1/4$;
- probability of one boy and one girl $P(BG) = P(GB) = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = 1/2$.

Denote $P(B)$ the probability of at least one child is a boy. Using the Bayes' theorem:

$$P(BG|B) = \frac{P(B|BG)P(BG)}{P(B)} = \frac{1 \cdot \frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

Problem 2

Assume our experiment is to choose a number from the set

$$\{1, 2, 3, ..., 50\}$$

(naturals not exceeding 50).
We consider the following events:

- A = the chosen number is divisible by 10;
- B = the chosen number is even.
- C = the chosen number is from the set 10, 11, 12, ..., 20.

Describe the following events:

a) $A \cap B$;

$$A \cap B = \{10, 20, 30, 40, 50\}$$

b) $C \setminus B$

$$C \setminus B = \{11, 13, 15, 17, 19\}$$

c) $A \cup C$

$$A \cup C = \{10, 11, 12, ..., 20, 30, 40, 50\}$$

d) $\overline{A} \cap C$

$$\overline{A} \cap C = \{11, 12, 13, 14, 15, 16, 17, 18, 19\}$$

Problem 3

A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If A is the event that a number less than 5 occurs on a single toss of the die, find $P(A)$.

$$P(\text{even}) = 2P(\text{odd}),\ P(\text{odd}) + P(\text{even}) = 1 \implies P(\text{odd}) = \frac{1}{3},\ P(\text{even}) = \frac{2}{3}$$

Assume the even and odd numbers are equality probably in their respective categories:

$$P(5) = \frac{1}{9},\ P(6) = \frac{2}{9}\ \therefore P(A) = 1 - P(6) - P(5) = 1 - \frac{2}{9} - \frac{1}{9} = \frac{2}{3}$$

Problem 4

Three men are seeking public office. Candidates A and B are given about the same chance of winning, but candidate C is given twice the chance of either A or B.

a) What is the probability that C wins?

$$P(A) = P(B) = 2P(C),\ P(A) + P(B) + P(C) = 1 \implies P(A) = P(B) = \frac{2}{5}$$

$$\therefore P(C) = \frac{1}{5}$$

b) What is the probability that A does not win?

$$P(\overline{A}) = 1 - P(A) = \frac{3}{5}$$

Problem 5

There are n socks, 3 of which are red, in a drawer. What is the value of n if when 2 of the socks are chosen randomly, the probability they are both red is 1/2?

The probaiblity of the first sock being red is $\frac{3}{n}$. The probability of the second sock also being red is $\frac{2}{n-1}$. Denote the event of both socks being red A.

$$P(A) = \frac{3}{n} \cdot \frac{2}{n-1} = \frac{6}{n^2 - n} = \frac{1}{2} \overset{n \geq 3}{\implies} n = 4$$

Problem 6

Delegates from 10 countries, including Russia, France, England, and the United States, are to be seated in a row. What is the probability that the French and English delegates are to be seated next to each other, and the Russian and U.S. delegates are not to be next to each other?

There are a total of $10! = 3628800$ cases for the 10 delegates to seat randomly.

If we group the French and English delegates, we are left with 9! cases of combinations, plus the 2! cases inside the French-English delegate group. Overall the number of cases where the English and French delegates are seated together is $9!2! = 725760$.

Now we can find the cases where the English and French are together and Russian an U.S. are together. That gives $8!2!2! = 161280$ cases. Therefore, $725760 - 161280 = 564480$ cases where French-English are together and Russian-U.S. are not together. If we consider all cases equally probable, the overall probability is $P = \dfrac{564480}{3628800} = \frac{7}{45} \approx 15.6\%$.

Problem 7

An urn contains 3 red and 7 black balls. Players A and B withdraw balls from the urn consecutively until a red ball is selected. Find the probability that A selects the red ball (A draws the first ball, then B, and so on. There is no replacement of the drawn balls).

We can calculate the probability of all the different cases where Player A gets a red ball, OR Player A draws a black ball, Player B draws black ball too up until Player A draws a red ball or there are no black balls left in the urn.

First try. If Player A gets red first try, probalbiity is $P_1 = \frac{3}{10}$
Second try. If Player A gets red second try, probalbiity is $P_2 = \frac{7}{10} \cdot \frac{6}{9} \cdot \frac{3}{8} = \frac{7}{40}$
Third try. If Player A gets red third try, probalbiity is $P_3 = \frac{7}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{1}{12}$
Fourth try. If Player A gets red fourth try, probalbiity is $P_4 = \frac{7}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{3}{4} = \frac{1}{40}$
If Player A doesnt get after fourth try, then there are only red balls remaining in the urn and Player B will certainly win.

$$\therefore P = P_1 + P_2 + P_3 + P_4 = \frac{3}{10} + \frac{7}{40} + \frac{1}{12} + \frac{1}{40} = \frac{7}{12} \approx 58.3\%$$

Problem 8

A shooter A hits the target with probability 0.7, a shooter B hits it with probability 0.8. Each shooter make one shot independently of others. What is the probability that

a) at least one shooter hits the target?

   Probability no shooter hits is $0.3 \cdot 0.2 = 0.06$, therefore the probability of at least one hit is $1 - 0.06 = 0.94 = 94\%$.

b) exactly one shooter hits the target?

Probability that A hits and B misses is $0.7 \cdot 0.2 = 0.14$, Probability the B hits and A misses is $0.8 \cdot 0.3 = 0.24$. Therefore probability that exactly one hits is $0.14 + 0.24 = 0.38 = 38\%$.

Problem 9

A pair of dice is rolled until a sum of either 5 or 7 appears. Find the probability that a 5 occurs first.

Probability of a pair of dice rolling to a sum of 7 is $\dfrac{6}{36} = \dfrac{1}{6}$, and the probability for a sum fo 5 is $\frac{4}{36} = \frac{1}{9}$.
Therefore, probability that neither happens is $1 - \dfrac{1}{6} - \dfrac{1}{9} = \dfrac{26}{36}$.
For the winning probability, we need the probability of either getting a 5, or getting neither AND getting 5 before 7 the next time. Since the probability of getting a five and getting neither five nor seven are mutually exclusive, we can add the probabilities. Denote the probability of getting 5 before 7 P.

$$P = \frac{1}{9} + \frac{26}{36}P \implies P = \frac{\frac{1}{9}}{1 - \frac{26}{36}} = \frac{2}{5}$$

Problem 10

From the deck of 32 cards randomly chosen 6 cards. What is the probability to have cards of all suits.

The number of selections is $n = \dbinom{32}{6} = 906192$.

We have two possible favourable cases: 3 cards of one suit and one of each suit, and 2 pairs of card in two suits and one of the other two suits. There are eight cards in each suit, for a 32 card deck.
3-1-1-1 distribution:

$$\binom{8}{3}\binom{8}{1}\binom{8}{1}\binom{8}{1}$$

Now we multiply this by the different combinations for the suit with 3 cards is selected, which is $\dbinom{4}{1}$,

$$\therefore n_1 = \binom{8}{3}\binom{8}{1}\binom{8}{1}\binom{8}{1}\binom{4}{1} = 114688$$

2-2-1-1 distribution:

$$\binom{8}{2}\binom{8}{2}\binom{8}{1}\binom{8}{1}$$

Similary, there are $\dbinom{4}{2}$ combinations of which two suits are "selected" to have two cards.

$$\therefore n_2 = \binom{8}{2}\binom{8}{2}\binom{8}{1}\binom{8}{1}\binom{4}{1} = 301056$$

Therefore, the total probability:

$$P = \frac{n_1 + n_2}{n}\frac{114688 + 301056}{906192} = \frac{128}{279} \approx 45.9\%$$