Digital version

ENGS121 Mechanics Section C - Homework 2

Mher Saribekyan A09210183

February 11, 2024

Problem 1.

Given three vectors $\vec{A}$, $\vec{B}$, $\vec{C}$. Their magnitudes are given in arbitrary units.

$$\vec{A} = \begin{bmatrix} x_A \\ y_A \end{bmatrix}, \vec{B} = \begin{bmatrix} x_B \\ y_B \end{bmatrix}, \vec{C} = \begin{bmatrix} x_C \\ y_C \end{bmatrix}$$

(a) Give the resultant vector in terms of components.

$$\text{Resultant vector is } \vec{d} = \begin{bmatrix} x_d \\ y_d \end{bmatrix} = \vec{A} + \vec{B} + \vec{C} = \begin{bmatrix} x_A + x_B + x_C \\ y_A + y_B + y_C \end{bmatrix}$$

(b) Give the resultant vector in terms of magnitude.

$$\left\|\vec{d}\right\| = \sqrt{(x_d)^2 + (y_d)^2}$$

(c) Give the angle with the +x axis.

$$\text{Angle between x axis } = \theta = \arctan\left(\frac{y_d}{x_d}\right)$$

$$\text{If } x_d \leq 0, y_d \leq 0 \implies \varphi = |\pi - \theta|$$

$$\text{If } x_d \leq 0, y_d \geq 0 \implies \varphi = \theta + \pi$$

(d) Determine the magnitude of $\vec{A} - \vec{C}$, show that it's the same as $\vec{C} - \vec{A}$.

$$\left\|\vec{A} - \vec{C}\right\| = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2} = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2} = \left\|\vec{C} - \vec{A}\right\|$$

(e) Determine the magnitude of $2\vec{A} - 4\vec{B} + 2\vec{C}$.

$$\left\|2\vec{A} - 4\vec{B} + 2\vec{C}\right\| = \sqrt{(2x_A - 4x_B + 2x_C)^2 + (2y_A - 4y_B + 2y_C)^2}$$

(f) Determine the magnitude of $\vec{A} - \vec{B} - \vec{C}$

$$\left\|\vec{A} - \vec{B} - \vec{C}\right\| = \sqrt{(x_A - x_B - x_C)^2 + (y_A - y_B - y_C)^2}$$

Problem 2.

A projectile is shot from the edge of a cliff 115m above ground level with an initial speed of 65.0ms at an angle of 35.0° with the horizontal.

(a) Determine the time taken by the projectile to hit point P at ground level.
Since the projectile goes upwards and then downwards, we can split the displacement into two parts, from where the projectile is shot to where it reaches the same level (t_1), and where the projectile falls from that level to the ground (t_2).
At starting point the velocity is v_0, while at the topmost point it is 0, because of a constant acceleration of g downwards.

$$g = \frac{\Delta v}{t} = \frac{v_0 \sin\alpha - 0}{t} \implies t_{\text{up}} = \frac{v_0 \sin\alpha}{g} = \frac{65.0 \cdot \sin(35.0°)}{9.8} = 3.8\text{s}$$

$$\text{Because of symmetry } t_1 = 2 \cdot t_{\text{up}} = 7.6\text{s}$$

$$S = v_0 \sin(\alpha) t_2 + \frac{g t_2^2}{2} \implies 115 = 65 \sin(35.0°) t_2 + \frac{9.8 \cdot t_2^2}{2}$$

$$\implies 4.9 t_2^2 + 37.3 t_2 - 115 = 0 \text{ and } t_2 > 0 \implies t_2 = 2.4\text{s} \implies t = t_1 + t_2 = 2.4 + 7.6 = 10\text{s}$$

(b) Determine the distance X of point P from the base of the vertical cliff. At the instant just before the projectile hits point P.

$$v_{\text{vertical}} = \text{constant} \implies S = v_0 \cos\alpha \cdot t = 65.0 \cos(35.0°) \cdot 10 = 532\text{m}$$

(c) Find the horizontal and the vertical components of its velocity.

$$v_{\text{vertical}} = v_{0\text{vertical}} + g t_2 = 65 \sin(35.0°) + 9.8 \cdot 2.4 = 61\text{m/s}$$

$$v_{\text{horizontal}} = \text{constant} = 65.0 \cos(35.0°) = 53\text{m/s}$$

(d) Find the magnitude of the velocity.

$$v = \sqrt{v_{\text{vertical}}^2 + v_{\text{horizontal}}^2} = \sqrt{61^2 + 53^2} = 81\text{m/s}$$

(e) Find the angle made by the velocity vector with the horizontal.

$$\varphi = \arctan\left(\frac{v_{\text{vertical}}}{v_{\text{horizontal}}}\right) = \arctan\left(\frac{61}{53}\right) = 49.0°$$

(f) Find the maximum height above the cliff top reached by the projectile.

$$H = h + v_0 \sin\alpha t_{\text{up}} - \frac{g t_{\text{up}}^2}{2} = 65 \sin(35.0°) 3.8 - \frac{9.8 \cdot 3.8^2}{2} = 186$$

Problem 3.

A fire hose held near the ground shoots water at a speed of 10m/s.

(a) At what angle(s) should the nozzle point in order that the water land 5 m away.

$$t = 2 \cdot \frac{v_0 \sin(\theta)}{g} \text{ and } S = v_0 cos(\theta) t = \frac{v_0^2 sin(2\theta)}{g}$$

$$\theta = \frac{1}{2} \sin\left(\frac{gS}{v_0^2}\right) = \frac{1}{2} \sin\left(\frac{9.8 \cdot 5}{10^2}\right) \implies \theta_1 = 14.7^\circ \text{ and } \theta_2 = 75.3^\circ$$

(b) Why are there two different angles?
The maximum distance an object can travel is when it is thrown at a 45° angle. From angles 0° to 45° the distance increases, while from 45° to 90° it decreases, which is why we have two angles in those two intervals.

(c) Sketch the two trajectories.