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ENGS121 Mechanics Section C - Homework 3

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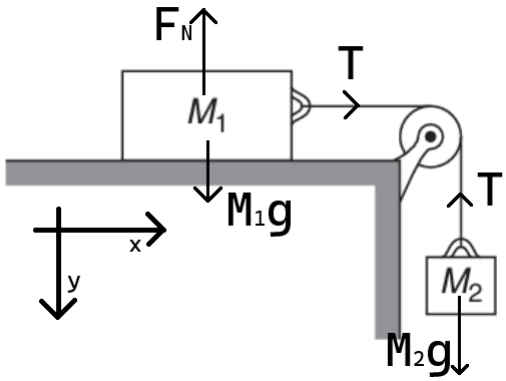
Problem 1 (30 points).

The two blocks M_1 and M_2 shown in the sketch are connected by a string of negligible mass. M_1 is $10kg$ and M_2 is $4kg$.

a) Draw the force diagrams.



b) What is the acceleration of the system. Neglect friction.
Apply Newton's second law.

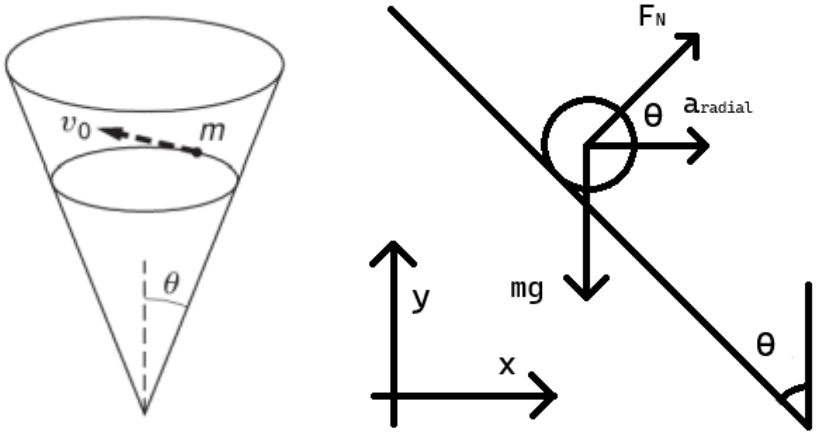
$$\begin{cases} T = M_1 a_{1x} \\ M_2 g - T = M_2 a_{2y} \end{cases} \overset{a_{1x}=a_{2y}:=a}{\Longrightarrow} M_2 g - M_1 a = M_2 a \implies a = \frac{M_2 g}{M_1 + M_2} = \frac{4 \cdot 9.8}{10 + 4} = 2.8 m/s^2$$

c) Find the tension of a string.

$$T = M_1 a_{1x} = 10 \cdot 2.8 = 28N$$

Problem 2 (20 points).

A particle of mass $m = 2kg$ slides without friction on the inside of a cone. The axis of the cone is vertical, and gravity is directed downward. The apex half-angle of the cone is $\theta = 45°$, as shown. The path of the particle happens to be a circle in a horizontal plane. The speed of the particle is $v_0 = 10m/s$. Draw a force diagram and find the radius of the circular path in terms of v_0, g and θ.



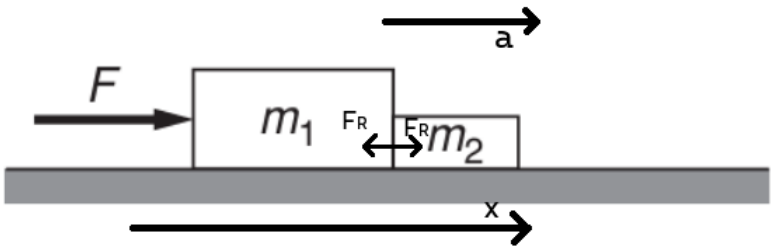
Apply Newton's second law.

$$\begin{cases} F_N \sin\theta - mg = ma_y = 0 \\ F_N \cos\theta = ma_x = ma_{radial} \end{cases} \implies \begin{cases} F_N \sin\theta = mg \\ F_N \cos\theta = m\frac{v_0^2}{r} \end{cases} \implies \tan\theta = \frac{gr}{v_0^2}$$

$$r = \frac{v_0^2 \tan\theta}{g} = \frac{10^2 \cdot 1}{9.8} = 10.2m$$

Problem 3 (15 points).

Two blocks m_1 and m_2 are in contact on a horizontal table. A horizontal force is applied to one of the blocks, as shown in the drawing. If $m_1 = 2kg$, $m_2 = 1kg$ and $F = 3N$, find the force of contact between the two blocks.



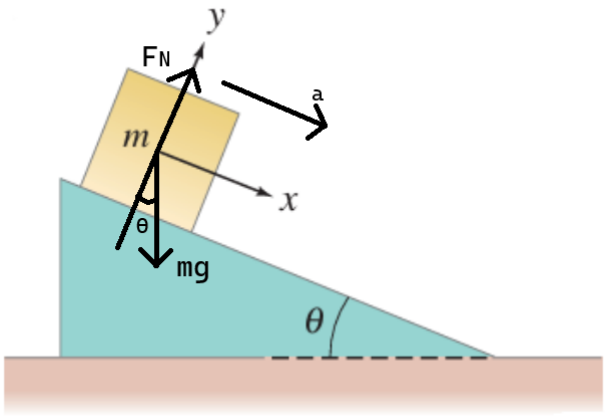
Since the problem does not explicitly require a force diagram, we can draw the forces only in the horizontal plane, as there is no movement in the vertical direction. The block m_1 exerts a force F_R on the block m_2 and by Newton's third law, the block m_2 exerts an equal and opposite force F_R on the block m_1. Now we use Newton's second law on both blocks separately. Since both blocks touch each other, the acceleration will be the same for both of the blocks.

$$\begin{cases} F - F_R = m_1 a \\ F_R = m_2 a \end{cases} \implies F = m_1 a + m_2 a \implies a = \frac{F}{m_1 + m_2}$$

$$F_R = m_2 a = \frac{F \cdot m_2}{m_1 + m_2} = \frac{3 \cdot 1}{2 + 1} = 1N$$

Problem 4 (25 points).

The block has mass m and lies on a fixed smooth frictionless plane tilted at an angle $\theta = 30.0°$ to the horizontal.



a) Determine the acceleration of the block as it slides down the plane.
Project the force on the x axis and use Newton's second law.

$$mg\sin\theta = ma \implies a = g\sin\theta = 9.8\sin 30.0° = 4.9m/s^2$$

b) If the block starts from rest $10.0m$ up the plane from its base, what will be the block's speed when it reaches the bottom of the incline?
Acceleration is $a = 4.9m/s^2$ and distance is $S = 10m$.

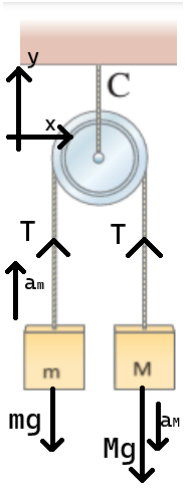
$$S = v_0 t + \frac{at^2}{2} \overset{v_0=0}{\implies} S = \frac{at^2}{2} \overset{t\geq 0}{\implies} t = \sqrt{\frac{2S}{a}}$$

$$a = \frac{v}{t} \implies v = at = a\sqrt{\frac{2S}{a}} = \sqrt{2Sa} \implies v = \sqrt{2\cdot 10\cdot 4.9} \approx 9.9m/s$$

c) Draw the force diagram.

Problem 5 (10 points).

Suppose the pulley is suspended by a cord C. Determine the tension in this cord after the masses ($m = 1kg$ and $M = 3kg$) are released and before one hits the ground. Ignore the mass of the pulley and cords.



Use Newton's second law on both masses on y axis. We can clearly see that both masses have the same acceleration, because they are tied together with a cord.

$$\begin{cases} T - mg = ma_m \\ T - Mg = -Ma_M \end{cases} \overset{a_m = a_M := a}{\Longrightarrow} -mg - (-Mg) = ma - (-Ma) \implies$$

$$\implies Mg - mg = Ma + ma \implies a = \frac{M-m}{M+m} g$$

$$T - mg = ma_m \overset{a_m = a}{\Longrightarrow} T = m(a+g) = m\left(\frac{M-m}{M+m} g + g\right) = mg\left(\frac{M-m}{M+m} + 1\right) = mg\frac{2M}{M+m}$$

$$T = 1 \cdot 9.8 \cdot \frac{2 \cdot 3}{1+3} = 14.7N$$