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ENGS121 Mechanics Section C - Homework 4

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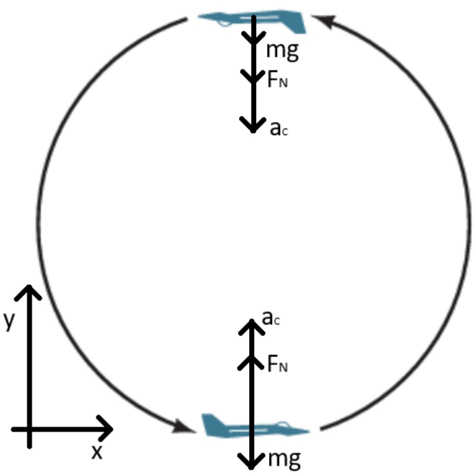
Problem 1 (30 points).

A jet pilot takes his aircraft in a vertical loop.



(a) If the jet is moving at a speed of $900km/h$ at the lowest point of the loop, determine the minimum radius of the circle so that the centripetal acceleration at the lowest point does not exceed $6.0g$-s.

$$a_{\text{centripetal}} = \frac{v^2}{r} \leq 6g \implies r_{\min} = \frac{v^2}{6g} = \frac{\left(\frac{900 \cdot 1000}{3600}\right)^2}{6 \cdot 9.8} \approx 1062m$$

(b) Calculate the $90kg$ pilot's effective weight (the force with which the seat pushes up on him) at the bottom of the circle (assume the same speed).

Since this sub-problem does not provide the data needed to calculate the centripetal force on the jet, the value of $6g$-s was assumed from point a) of this problem. Newton's second law was used on the y axis.

$$F_N - mg = F_{\text{centripetal}} \implies F_N = F_{\text{centripetal}} + mg = m6g + mg = 7mg = 6174N$$

(c) Calculate the $90kg$ pilot's effective weight (the force with which the seat pushes up on him) at the top of the circle (assume the same speed).

Since this sub-problem does not provide the data needed to calculate the centripetal force on the jet, the value of $6g$-s was assumed from point a) of this problem. Newton's second law was used on the y axis.

$$F_N + mg = F_{\text{centripetal}} \implies F_N = F_{\text{centripetal}} - mg = m6g - mg = 5mg = 4410N$$

Note: The normal force acting on the jet pilot at the top of the loop has the opposite direction to the normal force acting on the jet pilot at the bottom of the loop, as clearly depicted in the figure.

Problem 2 (10 points).

How large must the coefficient of static friction be between the tires and the road if a car is to round a level curve of radius $125m$ at a speed of $50.4km/h$?

$$F_{\text{centripetal}} = \frac{mv^2}{r}, N = mg \implies F_{\text{friction}} \leq \mu N = \mu mg$$

$$F_{\text{centripetal}} = F_{\text{friction}} \implies \frac{mv^2}{r} \leq \mu mg \implies \mu = \geq \frac{v^2}{gr}$$

$$\mu_{\text{min}} = \frac{\left(\frac{50.4 \cdot 1000}{3600}\right)^2}{9.8 \cdot 125} = 0.16$$

Problem 3 (20 points).

(a) What is the maximum speed with which a $1000kg$ car can round a turn of radius $100m$ on a flat road if the coefficient of friction between tires and road is the result acquired from the previous problem (problem 2)? (b) Is this result independent of the mass of the car?

Since this problem does not specify static or kinetic friction, it was assumed static, otherwise the car would slip and fall over.

$$\mu \geq \frac{v^2}{gr} \implies v \leq \sqrt{\mu gr} \implies v_{\text{max}} = \sqrt{0.16 \cdot 9.8 \cdot 100} = \sqrt{156.8} \approx 12.5m/s$$

Yes, the mass of the car does not have an impact on the maximum velocity cars can have on curves with the same radius driving with the same speed, because the lighter the car is, proportionally lighter the centripetal and the static friction forces are.

Problem 4 (20 points).

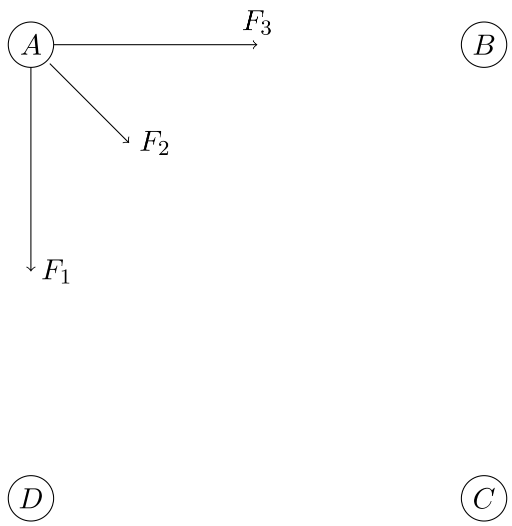
A particle revolves in a horizontal circle of radius $2m$. At a particular instant, its acceleration is $1m/s^2$ in a direction that makes an angle of $30°$ to its direction of motion. Determine its speed (a) at this moment, and (b) $2s$ later, assuming constant tangential acceleration.

$$a_{\text{centripetal}} = a \sin\varphi = \frac{v^2}{r} \implies v_0 = \sqrt{ar \sin\varphi} = \sqrt{1 \cdot 2 \cdot \sin 30°} = 1m/s$$

$$a_{\text{tangential}} = a \cos\varphi = 1 \cos 30° = \frac{\sqrt{3}}{2} = \frac{\Delta v}{t} \implies v = v_0 + at = 1 + \frac{\sqrt{3}}{2} \cdot 2 \approx 2.73m/s$$

Problem 5 (20 points).

Four $8kg$ spheres are located at the corners of a square of side $d = 2m$. Calculate (a) the magnitude and (b) direction of the gravitational force exerted on one sphere by the other three.



Consider the sphere at the point A: $F_1 = F_3 = G\dfrac{mm}{d^2}, F_2 = G\dfrac{mm}{(d\sqrt{2})^2} = G\dfrac{mm}{2d^2}$

$$F = F_2 + F_1 \cos 45^\circ + F_3 \cos 45^\circ = G\frac{mm}{2d^2} + G\frac{mm}{d^2} \cdot \frac{\sqrt{2}}{2} + G\frac{mm}{d^2} \cdot \frac{\sqrt{2}}{2}$$

$$= Gmm\left(\frac{1}{2d^2} + \frac{\sqrt{2}}{2d^2} + \frac{\sqrt{2}}{2d^2}\right) = G\frac{(2\sqrt{2}+1)mm}{2d^2} = 6.67 \cdot 10^{-11}\frac{(2\sqrt{2}+1) \cdot 8 \cdot 8}{2 \cdot 2^2} = 2.04 \cdot 10^{-9}N$$

The gravitational force of attraction on the sphere at the point A is directed from point A to point C.