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ENGS121 Mechanics Section C - Homework 6

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Exercise 1 (20 points).

A $800kg$ two-stage rocket is traveling at a speed of $v_0 = 5 \times 10^3 m/s$ away from Earth when a predesigned explosion separates the rocket into two sections of equal mass that then move with a speed of $v = 2\times10^3 m/s$ relative to each other along the original line of motion.

a) What is the speed of each section (relative to Earth) after the explosion?
After the explosion, the two parts do not change direction, only the first half gets a higher velocity than the second. The difference of these velocities is $2 \times 10^3 m/s$. v_1 was denoted for the velocity of the top part of the rocket, while v_2, for the bottom part. A formula for the conservation of momentum was applied.

$$p_0 = p_1 + p_2 \implies mv_0 = \frac{m}{2}v_1 + \frac{m}{2}v_2 \implies 2v_0 = v_1 + v_2 \text{ and we know that } v_1 - v_2 = v$$

Solving the following system of equation we get:

$$\begin{cases} v_1 + v_2 = 2v_0 \\ v_1 - v_2 = v \end{cases} \longrightarrow \begin{cases} v_1 = \frac{2v_0+v}{2} \\ v_2 = \frac{2v_0-v}{2} \end{cases} \longrightarrow \begin{cases} v_1 = 6 \times 10^3 m/s \\ v_2 = 4 \times 10^3 m/s \end{cases}$$

b) How much energy was supplied by the explosion?
By the first law of thermodynamics, the total energy of the system is conserved. The energy of the system before the explosion is the kinetic energy of the rocket and the potential energy that was used for the explosion. The energy of the system after the explosion are the kinetic energies of both rocket pieces. Hence the following equation is constructed and solved:

$$KE_0 + PE_{\text{explosion}} = KE_1 + KE_2 \implies \frac{mv_0^2}{2} + A_{\text{explosion}} = \frac{\frac{m}{2}v_1^2}{2} + \frac{\frac{m}{2}v_2^2}{2}$$

$$A_{\text{explosion}} = \frac{\frac{m}{2}v_1^2}{2} + \frac{\frac{m}{2}v_2^2}{2} - \frac{mv_0^2}{2} = \frac{\frac{800}{2}(6 \times 10^3)^2}{2} + \frac{\frac{800}{2}(4 \times 10^3)^2}{2} - \frac{800(5 \times 10^3)^2}{2} = 4\times10^8 J = 400MJ$$

Exercise 2 (20 points).

A $100kg$ fullback is running at $4m/s$ to the east and is stopped in $1s$ by a head-on tackle by a tackler running due west. Calculate

a) the original momentum of the fullback,

$$p_0 = mv = 100 \cdot 4 = 400Ns$$

b) the impulse exerted on the fullback,

$$\text{Since the fullback comes to a stop} \implies p_1 = 0 \implies \text{Impulse } = -400Ns$$

c) the impulse exerted on the tackler, and

$$\text{Due to the conservation of momentum} \implies p_1 = 0 \implies \text{Impulse } = 400Ns$$

d) the average force exerted on the tackler.
The definition of force was used. Force is defined as derivative of momentum.

$$F = \frac{\mathrm{d}p}{\mathrm{d}t} \approx \frac{\Delta p}{\Delta t} = \frac{400}{1} = 400N$$

Exercise 3 (20 points).

A $0.4kg$ croquet ball makes an elastic head-on collision with a second ball initially at rest. The second ball moves off with half the original speed of the first ball.

a) What is the mass of the second ball?
Elastic collision means that the energy was conserved. And from the term "head-on collision", it was assumed that the balls move in the same axis. The first ball was denoted with A, the second ball, B. It was given that $v_{A0} = 2v_{B1}$. The conservation of energy and momentum were used to constructed a system of equations:

$$\begin{cases} KE_{A0} + KE_{B0} = KE_{A1} + KE_{B1} \\ p_{A0} + p_{B0} = p_{A1} + p_{B1} \end{cases} \longrightarrow \begin{cases} \frac{m_A v_{A0}^2}{2} = \frac{m_A v_{A1}^2}{2} + \frac{m_B v_{B1}^2}{2} \\ m_A v_{A0} = m_A v_{A1} + m_B v_{B1} \end{cases}$$

$$\begin{cases} \frac{0.4 v_{A0}^2}{2} = \frac{0.4 v_{A1}^2}{2} + \frac{m_B v_{A0}^2}{8} \\ 0.4 v_{A0} = 0.4 v_{A1} + \frac{m_B v_{A0}}{2} \end{cases} \implies \begin{cases} v_{A1}^2 = v_{A0}^2 - \frac{5}{8} m_B v_{A0}^2 \\ m_B v_{A0} = 0.8 v_{A0} - 0.8 v_{A1} \end{cases}$$

$$\therefore \begin{cases} v_{A1} = v_{A0}\sqrt{1 - \frac{5}{8} m_B} \\ m_B \cancel{v_{A0}} = 0.8 \cancel{v_{A0}} - 0.8 \cancel{v_{A0}} \sqrt{1 - \frac{5}{8} m_B} \end{cases} \implies \left(1 - \frac{5}{8} m_B\right) = \left(1 - \frac{m_B}{0.8}\right)^2$$

$$1 - 0.625 m_B = 1 - 2.5 m_B + 1.5625 m_B^2 \implies 1.5625 m_B^2 - 1.875 m_B = 0$$

$$m_B(1.5625 m_B - 1.875) = 0 \implies m_B = \frac{1.875}{1.5625} = 1.2kg$$

b) What fraction of the original kinetic energy gets transferred to the second ball?

$$\frac{KE_{B1}}{KE_{A0}} = \frac{\frac{m_B v_{B1}^2}{2}}{\frac{m_A v_{A0}^2}{2}} = \frac{\frac{m_B v_{B1}^2}{2}}{\frac{m_A (2 v_{B1})^2}{2}} = \frac{m_B}{4 m_A} = \frac{1.2}{4 \cdot 0.4} = \frac{3}{4} = 0.75$$

Exercise 4 (20 points).

A $150g$ baseball moving $30m/s$ strikes a stationary $5kg$ brick resting on small rollers so it moves without significant friction. After hitting the brick, the baseball bounces straight back, and the brick moves forward at $1.2m/s$.

a) What is the baseball's speed after the collision?
The conservation of momentum was used. A is denoted for the baseball, while B was denoted for the brick. All units were converted to the base SI units.

$$p_{A0} + p_{B0} = p_{A1} + p_{B1} \implies m_A v_{A0} = -m_A v_{A1} + m_B v_{B1}$$

$$0.150 \cdot 30 = -0.150 \cdot v_{A1} + 5 \cdot 1.2 \implies v_{A1} = 10m/s$$

b) Find the total kinetic energy before and after the collision.
The formula for the kinetic energy of an object was used.

$$KE_0 = \frac{m_A v_{A0}^2}{2} = \frac{0.150 \cdot 30^2}{2} = 67.5J$$

$$KE_1 = \frac{m_A v_{A1}^2}{2} + \frac{m_B v_{B1}^2}{2} = \frac{0.150 \cdot 10^2}{2} + \frac{5 \cdot 1.2^2}{2} = 11.1J$$

Exercise 5 (20 points).

Billiard ball A of mass $m_A = 0.2kg$ moving with speed $v_A = 2.5m/s$ strikes ball B, initially at rest, of mass $m_B = 0.4kg$. As a result of the collision, ball A is deflected off at an angle of $30°$ with a speed $v'_A = 2m/s$.

a) Taking the x axis to be the original direction of motion of ball A, write down the equations expressing the conservation of momentum for the components in the x and y directions separately.
The positive part of the y axis was taken to be the side which the ball A was deflected. The angle the ball B was deflected, was denoted θ'_B, measured from the x axis.

$$\text{For the } x \text{ direction: } p_{A0x} + p_{B0x} = p_{A1x} + p_{B1x} \implies m_A v_A = m_A v'_A \cos(30°) + m_B v'_B \cos(\theta'_B)$$

$$\text{For the } y \text{ direction: } p_{A0y} + p_{B0y} = p_{A1y} + p_{B1y} \implies 0 = m_A v'_A \sin(30°) - m_B v'_B \sin(\theta'_B)$$

b) Solve these equations for the speed, v'_B, and angle, θ'_B, of ball B after the collision. Do not assume the collision is elastic.

$$\begin{cases} 0.2 \cdot 2.5 \cdot 0.5 = 0.4 v'_B \sin(\theta'_B) \\ 0.2 \cdot 2.5 = 0.2 \cdot 2\frac{\sqrt{3}}{2} + 0.4 \cdot v'_B \cos(\theta'_B) \end{cases} \longrightarrow \begin{cases} v'_B \sin(\theta'_B) = \frac{5}{8} \\ v'_B \cos(\theta'_B) = \frac{5-2\sqrt{3}}{4} \end{cases}$$

$$\therefore \tan(\theta'_B) = \frac{25 + 10\sqrt{3}}{26} \implies \theta'_B \approx 58.4° \text{ and } v'_B \approx 0.73m/s$$