Digital version

ENGS121 Mechanics Section C - Homework 7

Mher Saribekyan A09210183

April 23, 2024

Problem 1 (20 points).

The tires of a car make 64 revolutions as the car reduces its speed uniformly from $90km/h$ to $54km/h$. The tires have a diameter of $0.5m$. (a) What was the angular acceleration of the tires? If the car continues to decelerate at this rate, (b) how much more time is required for it to stop?
Solving this problem is a matter of using the formula for angular acceleration and substituting the values for the velocities with calculated angular velocities.

$$v_0 = 25m/s \implies \omega_0 = \frac{v_0}{r} = \frac{25}{0.25} = 100rad/s$$

$$v_1 = 15m/s \implies \omega_1 = \frac{v_1}{r} = \frac{15}{0.25} = 60rad/s$$

$$\omega_1^2 = \omega_0^2 + 2\alpha\theta \implies \alpha = \frac{\omega_1^2 - \omega_0^2}{2\theta} = \frac{60^2 - 100^2}{2 \cdot 2\pi \cdot 64} \approx -7.96rad/s^2$$

$$\alpha = \frac{\Delta\omega}{t} \implies t = \frac{\Delta\omega}{\alpha} = \frac{0 - 60}{-7.96} \approx 7.54s$$

Problem 2 (20 points).

A person exerts a horizontal force of $50N$ on the end of a door $100cm$ wide. What is the magnitude of the torque if the force is exerted (a) perpendicular to the door and (b) at a $45°$ angle to the face of the door?
The solution to this problem is a matter of using a formula.

$$\tau = lF\sin\theta \implies \tau_1 = 1 \cdot 50 \cdot \sin\left(\frac{\pi}{2}\right) = 50Nm \text{ and } \tau_2 = 1 \cdot 50 \cdot \sin(45°) = \frac{50\sqrt{2}}{2}Nm \approx 35.4Nm$$

Problem 3 (20 points).

A dad pushes tangentially on a small hand-driven merry-go-round and is able to accelerate it from rest to a frequency of $20rpm$ in $10s$. Assume the merry-go-round is a uniform disk of radius $3m$ and has a mass of $600kg$, and two children (each with a mass of $30kg$) sit opposite each other on the edge. Calculate the torque required to produce the acceleration, neglecting frictional torque. What force is required at the edge?

Similar to the first two problems, this problem was solved using the deifnition of moment of inertia and formulas for torque and force.

$$\omega = \frac{20 \cdot 2\pi}{60} = \frac{2\pi}{3} rad/s$$

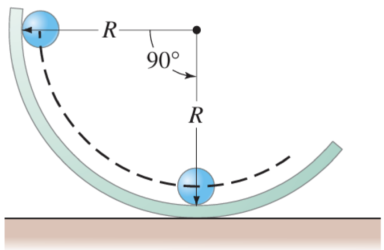
$$I = \sum m_i r_i^2 = \frac{Mr^2}{2} + 2 \cdot mr^2 = \frac{600 \cdot 3^2}{2} + 2 \cdot 30 \cdot 3^2 = 3240 kgm^2$$

$$\tau = I\alpha = I \cdot \frac{\Delta\omega}{t} = 3240 \cdot \frac{\frac{2\pi}{3} - 0}{10} = 216\pi \approx 679 Nm$$

$$F = \frac{\tau}{r} = \frac{216\pi}{3} = 72\pi \approx 226 N$$

Problem 4 (20 points).

A ball of radius $r = 1cm$ rolls on the inside of a track of radius $R = 100cm$. If the ball starts from rest at the vertical edge of the track, what will be its speed when it reaches the lowest point of the track, rolling without slipping?



The conservation of energy is used to solve this problem. At the top point there exists only potential energy, but at the bottom there exist both linear and rotational kinetic energies. The angular velocity of the ball is equal to the linear velocity divided by radius, hence $\omega = \frac{v}{r}$. The distance difference of the heights of the center of mass of the small ball at the bottom at top points is $R - r$.

$$E_{\text{total0}} = E_{\text{total1}} \implies mgh = \frac{mv^2}{2} + \frac{I\omega^2}{2} \implies \cancel{m}g(R-r) = \frac{\cancel{m}v^2}{2} + \frac{\frac{2}{5}\cancel{m}r^2\left(\frac{v}{r}\right)^2}{2}$$

$$v^2 = \frac{10}{7}g(R-r) \implies v = \sqrt{\frac{10}{7}g(R-r)} = \sqrt{\frac{10}{7} \cdot 9.8 \cdot (1 - 0.01)} \approx 3.72 m/s$$

Problem 5 (20 points).

(a) What is the angular momentum of a figure skater spinning at $5rev/s$ with arms in close to her body, assuming her to be a uniform cylinder with a radius of $20cm$, and a mass of $50kg$? (b) How much torque is required to slow her to a stop in $4s$, assuming she does not move her arms?
I don't want to imagine a girl being a cylinder, but the problem was solved with well-known formulas.

$$L = I\omega = \frac{mr^2}{2} \cdot \omega = \frac{50 \cdot 0.2^2}{2} \cdot 5 \cdot 2\pi = 10\pi \approx 31.4 kgm^2/s$$

$$\vec{L} = \sum \vec{r} \times \vec{p} \implies \frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\sum \vec{r} \times \vec{p} \implies \frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = \sum \frac{\mathrm{d}}{\mathrm{d}t}(\vec{r} \times \vec{p}) \overset{\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F}}{=} \sum \vec{r} \times \vec{F} = \vec{\tau}$$

$$\therefore \vec{\tau} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t} \implies \tau \approx \frac{\Delta L}{\Delta t} = \frac{0 - 31.4}{4 - 0} \approx -7.85 Nm$$