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ENGS121 Mechanics Section C - Homework 8

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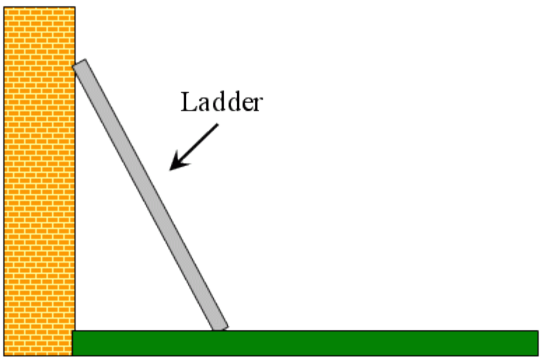
Problem 1 (20 points).

A uniform ladder of mass m and length l leans at an angle θ against a frictionless wall. If the coefficient of static friction between the ladder and the ground is μ_s, determine a formula for the minimum angle at which the ladder will not slip.

The status of equilibrium was used, suggesting that both net force and net torque are zero. Forces were projected in the vertical direction, to use that net force in the vertical direction is zero. Since the wall is frictionless, there is no vertical component of force. The top point of the ladder was used as a reference point for torques.



$$\begin{cases} mg = F_N \\ \frac{l}{2} mg\cos\theta + lF_{friction}\sin\theta = lF_N\cos\theta \end{cases} \implies \frac{1}{2} mg\cos\theta + \mu_s mg\sin\theta = mg\cos\theta$$

$$\therefore \frac{\cos\theta}{2} + \mu_s\sin\theta = \cos\theta \implies \tan\theta = \frac{1}{2\mu_s} \implies \theta = \arctan\frac{1}{2\mu_s}$$

Problem 2 (20 points).

A scallop forces open its shell with an elastic material called abductin, whose Young's modulus is about $2.0 \times 10^6 N/m^2$. If this piece of abductin is $4mm$ thick and has a cross-sectional area of $0.6cm^2$ how much potential energy does it store when compressed $2mm$?

The definition of work was used. The force was expressed in terms of Δl, which is the distance traveled S, in the definition of work. l_0 is the uncompressed position and is equal to 0, while l_1, is the $2mm$ compressed position.

$$\text{We know: } E = \frac{\sigma}{\varepsilon} = \frac{F}{A} \cdot \frac{l}{\Delta l} \implies F = \frac{E \cdot A \cdot \Delta l}{l} \text{ and } A = \int F \, \mathrm{d}S$$

$$E = \int_{l_0}^{l_1} \frac{E \cdot A \cdot \Delta l}{l} \, \mathrm{d}\Delta l = \frac{E \cdot A}{l} \left[\frac{(\Delta l)^2}{2} \right]_{l_0}^{l_1} = \frac{2.0 \times 10^6 \cdot 6.0 \times 10^{-5} \cdot (2 \cdot 10^{-3})^2}{2 \cdot 4 \cdot 10^{-3}} = 6.0 \times 10^{-2} J$$

Problem 3 (20 points).

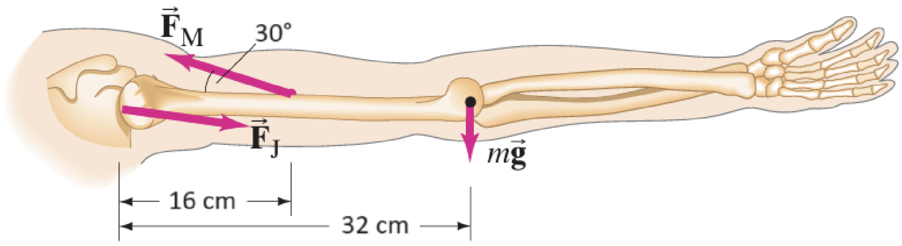
If a compressive force of $3.5 \times 10^4 N$ is exerted on the end of a $24cm$ long bone of cross-sectional area $3.5cm^2$, (a) will the bone break, and (b) if not, by how much does it shorten?

Ultimate compressive strength of a bone is $170 \times 10^6 N/m^2$. With the given data, the pressure on the bone is $\frac{3.5 \times 10^4}{0.00035} = 100 \times 10^6 N/m^2$, lower than the ultimate strength, so it won't fracture. We can use the Young's Modulus to calculate how much it was shortened.

$$E = \frac{\sigma}{\varepsilon} = \frac{F}{A} \cdot \frac{l}{\Delta l} \implies \Delta l = \frac{F \cdot l}{A \cdot E} = \frac{3.5 \times 10^4 \cdot 0.24}{0.00035 \cdot 15 \times 10^9} = 1.6 \times 10^{-3} m = 1.6mm$$

Problem 4 (20 points).

(a) Calculate the magnitude of the force, F_m, required of the "deltoid" muscle to hold up the outstretched arm shown in Fig. The total mass of the arm is $3.5kg$. (b) Calculate the magnitude of the force F_j exerted by the shoulder joint on the upper arm and the angle (to the horizontal) at which it acts.



We can use the fact that the net torque is zero. Hence:

$$mgL = F_M l \sin 30° \implies F_M = \frac{mgL}{l \sin 30°} = \frac{3.5 \cdot 9.8 \cdot 0.32}{0.16 \cdot 0.5} = 137.2N$$

If we use the fact that net force is zero, hence net force in the horizontal axis is also zero, hence the horizontal component of F_M is equal and opposite to that of F_j. Denote this axis x, and the vertical axis, y.

$$F_{jx} = -F_{Mx} = -F_M \cos 30° = -118.8N, \text{ and } F_{jy} = mg - F_{My} = mg - F_M \sin 30° = -34.3N$$

$$\left\| \vec{F_j} \right\| = \sqrt{(-34.3)^2 + (-118.8)^2} \approx 123.7N \text{ and } \varphi = \arctan \frac{-34.3}{-118.8} \approx 16.1°$$

Problem 5 (20 points).

A $M = 80kg$ adult sits at one end of a $l = 10m$ long board. His $m = 20kg$ child sits on the other end. (a) Where should the pivot be placed so that the board is balanced, ignoring the board's mass? (b) Find the pivot point if the board is uniform and has a mass of $m_b = 10kg$.

The solution to this problem is quite clear, after mentally imaging the described situation. Assume the distance from the adult to the pivot point is d. In a state of equilibrium, the net torque is zero. The pivot was taken as a reference point.

$$dMg = (l - d)mg \implies dM = lm - dm \implies d = \frac{m}{M + m}l = \frac{20}{80 + 20} \cdot 10 = 2m$$

If we take into account the mass of the board, we can also account for the torque created by its gravitational force, which is assumed to act from the center of gravity. The net torque formula is changed accordingly:

$$dMg = (l - d)mg + \left(\frac{l}{2} - d\right) m_b g \implies dM = lm - dm + \frac{lm_b}{2} - dm_b \implies d = \frac{m + 0.5m_b}{M + m + m_b}l = \frac{25}{11} \approx 2.3m$$