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ENGS123 Electricity and Magnetism - Homework 11

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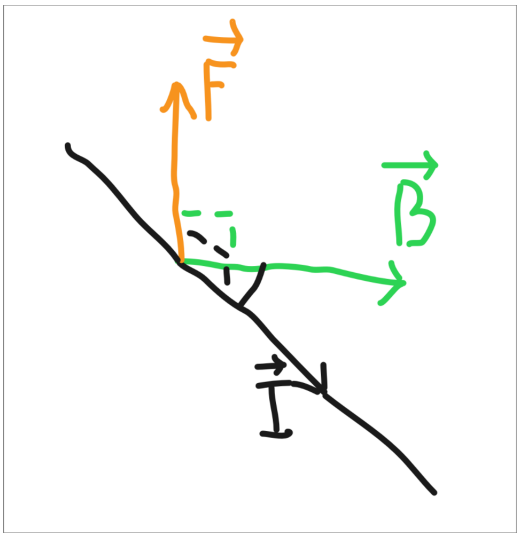
Problem 1

A straight wire is placed in a uniform magnetic field; the wire makes an angle of 30° with the magnetic field. The wire carries a current of $6.0A$, and the magnetic field has a strength of $0.40T$. Calculate the force on a $10cm$ segment of this wire. Show the direction of the force in a diagram.



$$\vec{F} = \vec{I}l \times \vec{B} \implies F = IBl\sin\varphi = 6 \cdot 0.1 \cdot 0.4 \cdot 0.5 = 0.12N$$

Problem 2

A straight segment of wire carries a current of $15A$ from the point $\vec{r_1} = 0.35\hat{\mathbf{i}} + 0.50\hat{\mathbf{j}}$ to the point $\vec{r_2} = 0.50\hat{\mathbf{i}} + 0.25\hat{\mathbf{j}} + 0.40\hat{\mathbf{k}}$, where distances are measured in meters. The wire segment is in a uniform magnetic field of $2.0T$ parallel to the positive z axis. What is the magnetic force on the wire?

$$\vec{I}l = \begin{bmatrix} 2.25 \\ -3.75 \\ 6 \end{bmatrix} Am, \vec{B} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} T \implies \vec{F} = \vec{I}l \times \vec{B} = \begin{bmatrix} -7.5 \\ -4.5 \\ 0 \end{bmatrix} N$$

Problem 3

An electromagnetic launcher, or a model for rail gun, consists of two parallel conducting rails across which is laid a conducting bar, which serves as a projectile. To launch this projectile, the rails are immersed in a magnetic field, and a current is sent through the rails and the bar (see Fig.). The magnetic force on the current in the bar then accelerates the bar (in actual rail guns of this kind, the magnetic field is itself produced by the current in the rails; but let us ignore this complication). Suppose that the magnetic field has a strength of $0.20T$ and the bar has a length of 0.10 m and a mass of 0.20 kg. Ignore friction. What current must you send through the bar to give it an acceleration of $1.0 \cdot 10^5 m/s^2$?

$$F = IlB = ma \implies I = \frac{ma}{lB} = \frac{0.2 \cdot 10^5}{0.1 \cdot 0.2} = 10^6 A$$

Problem 4

A horizontal circular loop of wire of radius $20cm$ carries a current of $25A$. At the location of the loop, the magnetic field of the Earth has a magnitude of $3.9 \cdot 10^{-5}T$ and points down at an angle of $16°$ with the vertical. What is the magnetic moment of this loop? What is the magnitude of the torque that this magnetic field exerts on the loop?

$$\mu = IS = 25 \cdot \pi \cdot (0.2)^2 \approx 3.14 Nm/T$$

$$\tau = \mu B \sin\varphi = \pi \cdot 3.9 \cdot 10^{-5} \cdot \sin(16°) \approx 3.38 \cdot 10^{-5} Nm$$

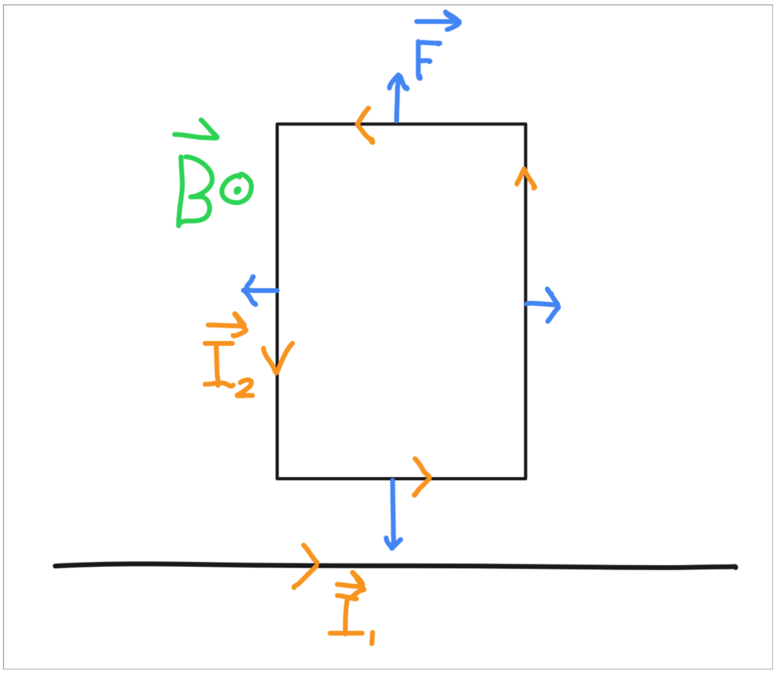
Problem 5

A rectangular loop of wire of dimensions $12cm \times 18cm$ is near a long, straight wire. The rectangle and the straight wire lie in the same plane. One of the short sides of the rectangle is parallel to the straight wire and at a distance of $6.0cm$; the long sides are perpendicular to the straight wire (Fig.). A current of $40A$ flows on the straight wire, and a current of $60A$ flows around the loop. What are the magnitude and direction of the net magnetic force that the straight wire exerts on the loop? Draw the force vector on each wire and the magnetic field lines near each wire.

$$\|F_{\text{bottom}}\| = IlB = 60 \cdot 0.12 \cdot \frac{4\pi \cdot 10^{-7} \cdot 40}{2\pi 0.06} \approx 9.6 \cdot 10^{-4} N$$

$$\|F_{\text{top}}\| = IlB = 60 \cdot 0.12 \cdot \frac{4\pi \cdot 10^{-7} \cdot 40}{2\pi 0.24} \approx 2.4 \cdot 10^{-4} N$$

$$\|F_{\text{sides}}\| = IlB = 60 \int_{0.06}^{0.24} \frac{4\pi \cdot 10^{-7} \cdot 40}{2\pi r} \,\mathrm{d}r \approx 6.65 \cdot 10^{-4} N$$



Problem 6

A circular loop of radius $r = 3.0cm$ has a total charge $Q = 5.0\mu C$ uniformly distributed around its circumference. The loop rotates about its axis with angular velocity $\omega = 6.3 \cdot 10^4 rad/s$. What is the magnetic moment of the loop?

$$\omega = \frac{2\pi r}{t} \implies I = \frac{Q}{t} = \frac{Q\omega}{2\pi r}, \mu = IS = \frac{Q\omega r}{2} = \frac{5 \cdot 10^{-6} \cdot 6.3 \cdot 10^4 \cdot 0.03}{2} = 4.725 \cdot 10^{-3} Nm/T$$

Problem 7

A Hall sensor made from silicon is $0.30mm$ thick and has a charge carrier density of $2.0 \times 10^{19} m^{-3}$. When it is placed in an unknown perpendicular magnetic field and carries a current of $5.0mA$ along its length, a Hall voltage of $130mV$ is measured across its width. What is the value of the magnetic field?

$$V_H = \frac{IB}{neL} \implies B = \frac{V_H neL}{I} = \frac{0.13 \cdot 2 \cdot 10^{19} \cdot 1.602 \cdot 10^{-19} \cdot 0.0003}{0.005} \approx 25.0mT$$