



Digital version

ENGS123 Electricity and Magnetism - Homework 12

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Problem 1

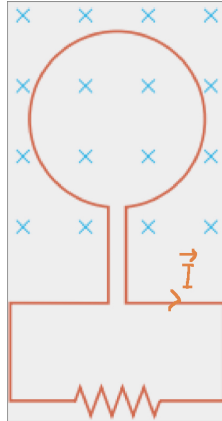
A circular coil with 250 turns of 1.0mm diameter copper wire has an area of $0.35m^2$. A magnetic field directed at 30° with respect to the plane of the coil increases uniformly from zero to $5.5T$ in $35s$. What is the EMF induced across the coil during this time? If the ends of the coil are connected, what current flows? How much electrical energy is dissipated during this time?

$$|\mathcal{E}| = \frac{d\Phi}{dt} = \frac{250 \cdot 0.35 \cdot 5.5 \sin(30^\circ)}{35} = 6.875V, 0.35 = \pi r^2 \implies r \approx 0.334m$$

$$I = \frac{V}{R} = \frac{6.875}{1.72 \cdot 10^{-8} \frac{500\pi \cdot 0.334}{\pi \cdot 0.0005^2}} \approx 0.6A, A = VIt \approx 4.11J$$

Problem 2

A circular loop of wire is placed in a magnetic field of 0.30 T while the free ends of the wire are attached to a 15Ω resistor as shown in Fig. When you squeeze the loop, the area of the loop is reduced at a constant rate from 200 to $100cm^2$ in $0.020s$. What are the magnitude and direction of the current in the resistor?



$$|\mathcal{E}| = \frac{d\Phi}{dt} \implies I = \frac{V}{R} = \frac{0.30(0.02 - 0.01)}{15 \cdot 0.020} = 10mA$$

Problem 3

A long rectangular conducting loop of width 25cm is partially in a region of a horizontal magnetic field of 1.8T perpendicular to the loop, as shown in Fig. The mass of the loop is 12g , and its resistance is 0.17Ω . If the loop is released, what is its terminal velocity? Assume that the top of the loop stays in the magnetic field.

$$I = \frac{V}{R} = \frac{\frac{d\Phi}{dt}}{R} = \frac{Bw\frac{dl}{dt}}{R} = \frac{Bwv}{R}$$

$$F = BIw = \frac{(Bw)^2v}{R} = mg \implies v = \frac{mgR}{(Bw)^2} = \frac{0.012 \cdot 9.81 \cdot 0.17}{(1.8 \cdot 0.25)^2} \approx 98.8\text{cm/s}$$

Problem 4

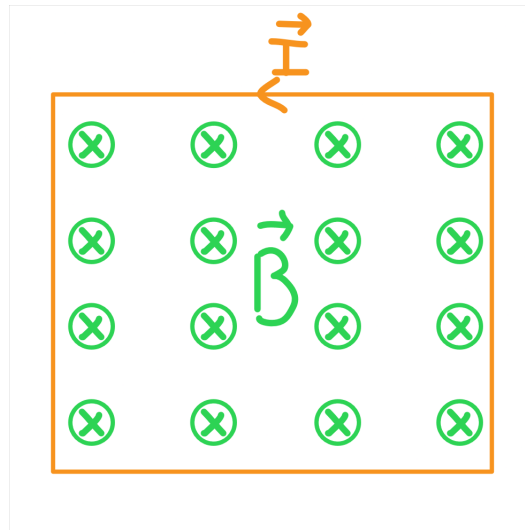
A metal circular wire with 33Ω resistance is 12cm in diameter. The disk is rotated at 300rev/s while in a 5.5T magnetic field perpendicular to the circular axis of rotation. Initially the magnetic field was perpendicular to the plane of the circle. What is the dependence of the current through the wire on time? What torque must be supplied to the disk to maintain this current?

$$I(t) = \frac{V(t)}{R} = \frac{B\omega S \sin(\omega t)}{R} = \frac{5.5 \cdot 600\pi \cdot 0.06^2\pi \sin(600\pi t)}{33} \approx 3.55 \sin(600\pi t)$$

$$\tau = BIl d = 5.5 \cdot 3.55 \cdot 2\pi \cdot 0.06 \cdot 0.12 \approx 0.88\text{Nm}$$

Problem 5

A square loop of dimension $8.0\text{cm} \times 8.0\text{cm}$ is made of copper wire of radius 1.0mm . The loop is placed face on in a magnetic field which is increasing at the constant rate of 80T/s . What induced current will flow around the loop? Draw a diagram showing the direction of the field and the induced current.



$$I = \frac{V}{R} = \frac{(0.08)^2 \cdot 80}{1.72 \cdot 10^{-8} \frac{4 \cdot 0.08}{\pi 0.001^2}} \approx 292\text{A}$$

Problem 6

A rectangular loop measuring $20\text{cm} \times 80\text{cm}$ is made of heavy copper wire of radius 0.13cm . Suppose you shove this loop, short side first, at a speed of 0.40m/s into a magnetic field of $5.0 \times 10^{-2}\text{T}$. The rectangle is face on to the magnetic field, and the trailing short side remains outside the magnetic field (see Fig.). What induced current will flow around the loop?

Using formula derived in problem 3:

$$I = \frac{Bwv}{\rho \frac{l}{S}} = \frac{5.0 \cdot 10^{-2} \cdot 0.20 \cdot 0.40}{1.72 \cdot 10^{-8} \frac{2}{\pi 0.0013^2}} \approx 0.62\text{A}$$

Problem 7

A metal rod of length l and mass m is free to slide, without friction, on two parallel metal tracks. The tracks are connected at one end so that they and the rod form a closed circuit Fig. The rod has a resistance R , and the tracks have negligible resistance. A uniform magnetic field is perpendicular to the plane of this circuit. The magnetic field is increasing at a constant rate of dB/dt . Initially the magnetic field has strength B_0 and the rod is at rest at a distance x_0 from the connected end of the rails. Express the acceleration of the rod at this instant in terms of the given quantities.

$$I = \frac{V}{R} = \frac{\frac{\text{d}\Phi}{\text{d}t}}{R} = \frac{S \frac{\text{d}B}{\text{d}t} + B \frac{\text{d}S}{\text{d}t}}{R} = \frac{x l \frac{\text{d}B}{\text{d}t} + (B_0 + \frac{\text{d}B}{\text{d}t} t) \frac{\text{d}S}{\text{d}t}}{R}, F = BIl = ma$$
$$\therefore \ddot{x} = a = \frac{BIl}{m} = \left(B_0 + \frac{\text{d}B}{\text{d}t} t \right) l \frac{x l \frac{\text{d}B}{\text{d}t} + (B_0 + \frac{\text{d}B}{\text{d}t} t) \frac{\text{d}S}{\text{d}t}}{mR}, x_0 \text{ is the initial condition of } x$$

Problem 8

In a fast digital circuit, the timing of signals is often limited by the inductance of circuit components. Suppose that a 5.0V EMF is suddenly applied to an effective inductance of 2.5H . How long does it take for the current in the inductor to reach 2.0mA ?

$$|\mathcal{E}| = L \frac{\text{d}I}{\text{d}t} \implies t = \frac{2.5 \cdot 0.002}{5} = 0.001\text{s} = 1\text{ms}$$

Problem 9

A ring of thick wire has a self-inductance of $4.0 \cdot 10^{-8}\text{H}$. How much work must you do to establish a current of 25A in this ring?

$$A = \frac{LI^2}{2} = \frac{4.0 \cdot 10^{-8} \cdot 25^2}{2} = 1.25 \cdot 10^{-5}\text{J}$$

Problem 10

A superconducting solenoid carries a current of 55A , has an inductance of 35H , and produces a magnetic field of 9.0T . What energy is stored in the solenoid? What is the volume of the solenoid?

$$E = \frac{35 \cdot 55^2}{2} = 52937.5\text{J}$$

$$L = N\mu_0 V \implies V = \frac{L}{N\mu_0} = \frac{35}{4\pi \cdot 10^{-7}} \approx 2.79 \cdot 10^7 m^3$$

Problem 11

The resistance of the coils of an electric motor is 2.0Ω when the motor is not in operation (not rotating). When the motor is connected to $115V$ and is rotating at full speed, it draws a current of $0.10A$. Deduce the back EMF that the coils of the motor produce when rotating.

$$V = 2 \cdot 0.1 = 0.2V \implies EMF = 114.8V$$

Problem 12

A long solenoid has 2000 turns per meter and a radius of $2.0cm$. What is the self-inductance for a $1.0m$ segment of this solenoid? What back EMF will this segment generate if the current in the solenoid is changing at the rate of $3.0 \cdot 10^{-2} A/s$?

$$L = \frac{2000^2 \cdot 4\pi \cdot 10^{-7} \cdot \pi \cdot 0.02^2}{1} \approx 6.32mH$$

$$\mathcal{E} = -L \frac{dI}{dt} \implies |\mathcal{E}| = 6.32 \cdot 10^{-3} \cdot 3.0 \cdot 10^{-2} \approx 0.19mV$$