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ENGS123 Electricity and Magnetism - Homework 1

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December 11, 2024

Problem 1

A plastic rod that has been charged to $-15.0nC$ touches a metal sphere. After touching, the rod's charge is $-10.0nC$.

a) What kind of charged particle was transferred between the rod and the sphere, and in which direction? That is, did it move from the rod to the sphere or from the sphere to the rod?

The charged particle that transferred betwee n the rod and the sphere was an electron, and it moved from the rod to the metal sphere, because an electron has a negative charge, and the charge in the plastic rod increased after the interaction, meaning it has lost electrons.

b) How many charged particles were transferred?

$$e \approx -1.602 \times 10^{-19} C \implies n_{\text{electron}} = \frac{-5 \times 10^{-9}}{-1.602 \times 10^{-19}} \approx 3.12 \times 10^{10}$$

Problem 2

Consider two protons separated by a distance of $1.0 \times 10^{-12} m$.

a) What is the gravitational force of attraction between the protons?

$$F_g = G\frac{m_1 m_2}{r^2} = 6.67 \times 10^{-11} \frac{\left(1.67 \times 10^{-27}\right)^2}{\left(1.0 \times 10^{-12}\right)^2} \approx 1.86 \times 10^{-40} N$$

b) What is the electric force of repulsion between these protons? What is the ratio of the electric force and the gravitational force?

$$|F_e| = k\frac{|q||Q|}{r^2} = 8.99 \times 10^9 \frac{\left(1.60 \times 10^{-19}\right)^2}{\left(1.0 \times 10^{-12}\right)^2} \approx 2.31 \times 10^{-4} N$$

$$\frac{F_e}{F_g} = 1.12 \cdot 10^{36}$$

c) Should we take into account the gravitational force then studying the net interaction force between two protons?

Since the ratio is so small, we can consider the gravitational interaction force insignificant compared to the electric interaction force, and not take into account during our calculations.

Problem 3

Suppose that you remove all the electrons in a copper penny of mass $2.7g$ and place them at a distance of 2.0 m from the remaining copper nuclei. What is the electric force of attraction on the electrons?

$$Z_{Cu} = 29 \text{ and } M_{Cu} = 63.546 \implies n_{\text{electrons}} = n_{\text{protons}} = 29 \cdot \frac{2.7}{63.645} \cdot 6.022 \times 10^{23} \approx 7.4086 \times 10^{23}$$

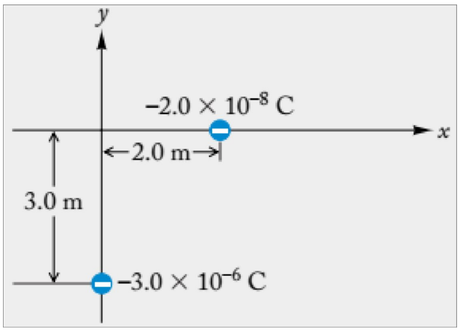
Because neutrons do not possess an electric charge, we can disregard their effect in the electric force of attraction.

$$q_{\text{protons}} = -q_{\text{electrons}} = 1.602 \times 10^{-19} \cdot 7.4086 \times 10^{23} \approx 1.187 \times 10^{5} C$$

$$F = -k\frac{q_{\text{electrons}} \cdot q_{\text{protons}}}{r^2} = 8.99 \times 10^9 \frac{\left(1.187 \times 10^5\right)^2}{2^2} \approx 3.17 \times 10^{19} N$$

Problem 4

A small charge of $-2.0 \times 10^{-8}C$ is at the point $x = 2.0m$, $y = 0m$ on the x axis. A second small charge of $-3.0 \times 10^{-6}C$ is at the point $x = 0m$, $y = -3.0m$ on the y axis (see Fig). What is the electric force that the first charge exerts on the second? What is the force that the second charge exerts on the first? Express your answers as vectors, with x and y components.



$$r = \sqrt{2^2 + 3^2} = \sqrt{13} m$$

$$|F| = k\frac{|q_1||q_2|}{r^2} = 8.99 \times 10^9 \frac{\left(2 \times 10^{-8}\right)\left(3 \times 10^{-6}\right)}{\sqrt{13}^2} \approx 4.149 \times 10^{-5} N$$

Because they both have negative charges, the exerted force on one charge will be away from the other charge. If we use Newton's third law on x and y project, we see that the x components of the forces will be the same, but in different directions, similarly for the y components. Denote F_1 the force the second charge exerts on the first charge and F_2 the force the first charge exerts on the second charge.

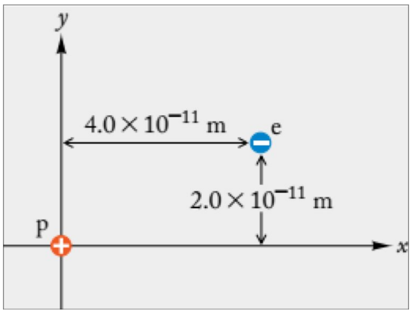
$$F_{1x} = |F|\cos\theta = 4.149 \times 10^{-5} \frac{2}{\sqrt{13}} \approx 2.30 \times 10^{-5} N \implies F_{2x} = -2.30 \times 10^{-5} N$$

$$F_{1y} = |F|\sin\theta = 4.149 \times 10^{-5} \frac{3}{\sqrt{13}} \approx 3.45 \times 10^{-5} N \implies F_{2y} = -3.45 \times 10^{-5} N$$

$$\therefore \vec{F_1} = \begin{bmatrix} 2.30 \\ 3.45 \end{bmatrix} \times 10^{-5} N \text{ and } \vec{F_2} = \begin{bmatrix} -2.30 \\ -3.45 \end{bmatrix} \times 10^{-5} N$$

Problem 5

A proton is at the origin of coordinates. An electron is at the point $x = 4.0 \times 10^{-11}m$, $y = 2.0 \times 10^{-11}$ in the $x - y$ plane (see Fig.). What are the x and y components of the electric force that the proton exerts on the electron? That the electron exerts on the proton?



$$r = \sqrt{20} \times 10^{-11}m \implies |F| = k\frac{|q_1||q_2|}{r^2} = 8.99 \times 10^9 \frac{\left(1.602 \times 10^{-19}\right)^2}{\left(\sqrt{20} \times 10^{-11}\right)^2} \approx 1.154 \times 10^{-7}N$$

Because they have opposite charges, they will attract each other. If we use Newton's third law on x and y project, we see that the x components of the forces will be the same, but in different directions, similarly for the y components. Denote F_e the force the proton exerts on the electron and F_p the force the electron exerts on the proton.

$$F_{px} = |F|\cos\theta = 1.154 \times 10^{-7}\frac{4}{\sqrt{20}} \approx 1.03 \times 10^{-7}N \implies F_{ex} = -1.03 \times 10^{-7}N$$

$$F_{py} = |F|\sin\theta = 1.154 \times 10^{-7}\frac{2}{\sqrt{20}} \approx 0.52 \times 10^{-7}N \implies F_{ey} = -0.52 \times 10^{-7}N$$

$$\therefore \vec{F_p} = \begin{bmatrix} 1.03 \\ 0.52 \end{bmatrix} \times 10^{-7}N \text{ and } \vec{F_e} = \begin{bmatrix} -1.03 \\ -0.52 \end{bmatrix} \times 10^{-7}N$$

Problem 6

Precise experiments have established that the magnitudes of the electric charges of an electron and a proton are equal to within an experimental error of $\pm 10^{-21}e$ and that the electric charge of a neutron is zero to within $\pm 10^{-21}e$. Making the worst possible assumption about the combination of errors, what is the largest conceivable electric charge of an oxygen atom consisting of 8 electrons, 8 protons, and 8 neutrons? Treating the atoms as point particles, compare the electric force between two such oxygen atoms with the gravitational force between these atoms. Is the net force attractive or repulsive?

We have similar cases, with only the sign being different. Without loss of generality, assume worst possible case that $|q_{\text{neutron}}| = -10^{-21}e$, and $|q_{\text{proton}}| - |q_{\text{electron}}| = -10^{-21}e$.

$$\therefore q_{\text{oxygen}} = -16 \times 10^{-21}e \approx 2.56 \times 10^{-39}C \text{ and } m_{\text{oxygen}} = 16u$$

$$F_g = G\frac{m_{\text{oxygen}}^2}{r^2} = 6.67 \times 10^{-11}\left(\frac{16}{6.022 \times 10^{23}}\right)^2 \cdot \frac{1}{r^2} \approx 4.71 \times 10^{-56} \cdot r^{-2}N$$

$$F_e = k\frac{q_{\text{oxygen}}^2}{r^2} = 8.99 \times 10^9 \left(2.56 \times 10^{-39}\right)^2 \cdot \frac{1}{r^2} \approx 5.89 \times 10^{-68} \cdot r^{-2}N$$

With the worst possible assumption, the electric fore is approximately 10^{12} times less than the gravitational force.
Since two such oxygen atoms will have the same positive (negative) charge, they will repel each other.

Problem 7

Under the influence of the electric force of attraction, the electron in a hydrogen atom orbits around the proton on a circle of radius $5.3 \times 10^{-11} m$. What is the orbital speed? What is the orbital period?

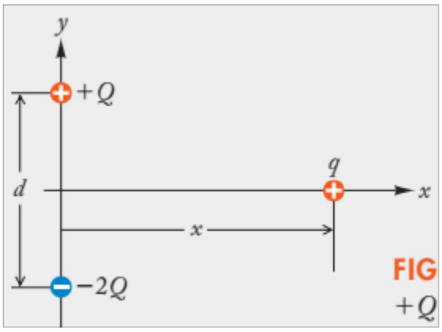
$$F_e = k\frac{e^2}{r^2} = \frac{m_e v^2}{r} = F_{\text{centripetal}} \implies v = \sqrt{\frac{ke^2}{m_e r}}$$

$$v = \sqrt{\frac{8.99 \times 10^9 \cdot (1.602 \times 10^{-19})^2}{9.11 \times 10^{-31} \cdot 5.3 \times 10^{-11}}} \approx 2.19 \times 10^6 m/s$$

$$T = \frac{2\pi r}{v} = \frac{2\pi \cdot 5.3 \times 10^{-11}}{2.19 \times 10^6} = 1.52 \times 10^{-16} s$$

Problem 8

Point charges $+Q$ and $-2Q$ are separated by a distance d. A point charge q is equidistant from these charges, at a distance x from their midpoint (see Fig.). What is the electric force on q? Find the x and y projections of that force.



$$|F_{Q\to q}| = k\frac{Qq}{r^2} \text{ and is directed from Q to q (they repel)}$$

$$|F_{-2Q\to q}| = k\frac{2Qq}{r^2} \text{ and is directed from q to -2Q (they attract)}$$

The y projections of the forces are summed and point to the $-y$ direction, while the x projects have opposite directions, so they are subtracted from each other, and it will point to the $-x$ direction, since the force of attraction is stronger, than the repelling force. Distance is $r = \sqrt{x^2 + \left(\frac{d}{2}\right)^2} = \frac{1}{2}\sqrt{4x^2 + d^2}$

$$F_x = 4k\frac{Qq}{4x^2 + d^2}\left(\frac{2x}{\sqrt{4x^2 + d^2}} - 2\frac{2x}{\sqrt{4x^2 + d^2}}\right) \text{ and } F_y = 4k\frac{Qq}{4x^2 + d^2}\left(\frac{d}{\sqrt{4x^2 + d^2}} + 2\frac{d}{\sqrt{4x^2 + d^2}}\right)$$

$$\vec{F} = \begin{bmatrix} -\frac{8kQqx}{\sqrt{(4x^2+d^2)^3}} \\ \frac{12kQqd}{\sqrt{(4x^2+d^2)^3}} \end{bmatrix}$$

Problem 9

A thin rod of length L is placed near a point charge q (see Fig.), with the nearest end a distance d from the charge and oriented radially as shown. The rod carries a uniform distribution of charge λ coulombs per meter. Find the electric force that acts on the rod.

Let's divide the rod into small parts with length $\mathrm{d}s$. The charge on that small part will be $\mathrm{d}Q = \lambda\,\mathrm{d}s$. The force acting on that small part from the point charge will be $\mathrm{d}F = k\dfrac{q\,\mathrm{d}Q}{r^2} = \dfrac{kq\lambda}{s^2}\,\mathrm{d}s$. Now if we sum up all those tiny forces acting on the rod, we get:

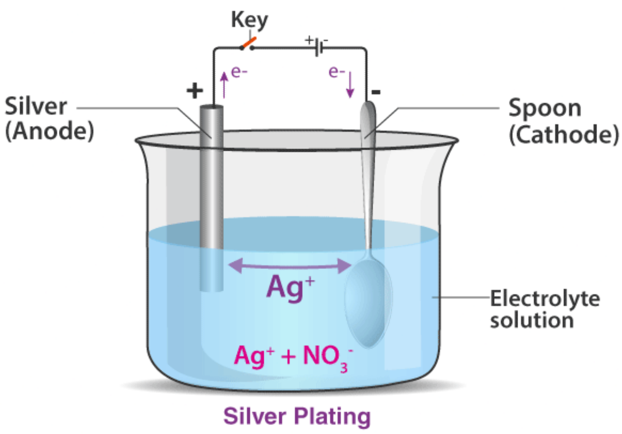
$$F = \int_d^{d+L} \frac{kq\lambda}{s^2}\,\mathrm{d}s = kq\lambda\left[-s^{-1}\right]_d^{d+L} = kq\lambda\left(\frac{1}{d} - \frac{1}{d+L}\right) = \frac{kq\lambda L}{d(d+L)}$$

Problem 10

We can silver-plate a metallic object, such as a spoon, by immersing the spoon and a bar of silver (Ag) in a solution of silver nitrate ($AgNO_3$). If we then connect the spoon and the silver bar to an electric generator and make a current flow from one to the other, the following reactions will occur at the immersed surfaces (Fig.):

$$Ag_{(\mathrm{metal})} \rightarrow Ag^{+} + [\mathrm{electron}]$$
$$Ag^{+} + [\mathrm{electron}] \rightarrow Ag_{(\mathrm{metal})}$$

The first reaction deposits silver on the spoon, and the second removes silver from the silver bar. How many electrons must we make flow from the silver bar to the spoon in order to deposit $1.0g$ of silver on the spoon?



For a single silver atom to deposit on the spoon, we need one electron to transfer from the anode to cathode. For $1g$, we would need $n_{\mathrm{electron}} = \dfrac{1}{107.9} \cdot 6.022 \times 10^{23} \approx 5.58 \times 10^{21}$ electrons to be moved.