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ENGS123 Electricity and Magnetism - Homework 2

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Problem 1

Millikan's experiment measures the elementary charge e by the observation of the motion of small oil droplets in an electric field. The oil droplets are charged with one or several elementary charges, and if the (vertical) electric field has the right magnitude, the electric force on the droplet will balance its weight, holding the drop suspended in midair. Suppose that an oil droplet of radius $1.0 \cdot 10^{-4} cm$ carries a single elementary charge. What electric field is required to balance the weight? The density of oil is $\rho = 0.80 g/cm^3$.

The term balance, means that the forces acting on the oil droplet cancel each other. Therefore, the acceleration on the oil droplet is zero and it will keep moving with a constant velocity.

$$E = \frac{F_E}{q}, F_G = mg, \text{ by Newton's second law } F_E = F_G \implies E = \frac{mg}{q}$$

Convert the mass into its SI unit kg, by multiplying with 10^{-3}.

$$E = \frac{V \rho g}{e} = \frac{\frac{4}{3}\pi (1.0 \cdot 10^{-4})^3 \cdot 0.80 \cdot 10^{-3} \cdot 9.8}{1.602 \cdot 10^{-19}} \approx 2.05 \cdot 10^5 N/C$$

Problem 2

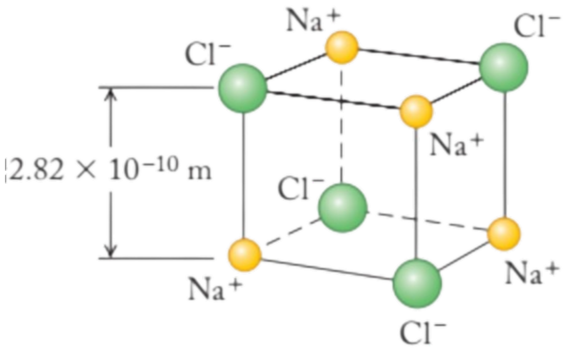
Electric breakdown (sparks) occurs in air if the electric field reaches $3.0 \cdot 10^6 N/C$. At this field strength, free electrons present in the atmosphere are quickly accelerated to such large speeds that upon impact on atoms they knock electrons off the atom and thereby generate an avalanche of electrons. How far must a free electron move under the influence of the above electric field if it is to attain a kinetic energy of $3.0 \cdot 10^{-19} J$ (which is sufficient to produce ionization)?

A uniform electric field was assumed in the solution of the problem.

$$A = FS = EqS \implies S = \frac{A}{Eq} = \frac{3.0 \cdot 10^{-19}}{3.0 \cdot 10^6 \cdot 1.602 \cdot 10^{-19}} = 6.24 \cdot 10^{-7} m$$

Problem 3

Consider eight of the ions of Cl^- and Na^+ on a crystal lattice of common salt. The ions are located at the vertices of a cube measuring $2.82 \cdot 10^{-10} m$ on an edge (Fig.). Calculate the magnitude of the electric force that seven of these ions exert on the eighth.



A Chlorine ion has one extra electron, meaning it has the same charge as an electron. A Sodium ion has one less electron, meaning it has the same charge as a proton, which is the same absolute charge is the electron. Since all ions have the same absolute charge, it does not matter which one we choose to do the calculations on. First, we calculate the force of attraction to the nearby ions, adjacent to the ion. As those three forces are orthogonal to each other, we can calculate the sum of those vectors, which will be in the direction of the diagonal and with a magnitude $\sqrt{3}$ times one of the forces. Same procedure can be carried out for the three likewise charged ions, but the force will be a repelling one with an opposite direction to the other three forces. Those three ions are distance $\sqrt{2}$ times further than than the other three. Meanwhile, the remaining one ion is $\sqrt{3}$ times further than the first group of ions.

$$F_{123} = \sqrt{3}k\frac{e^2}{d^2} = 8.99 \cdot 10^9 \frac{\left(1.602 \cdot 10^{-19}\right)^2}{\left(2.82 \cdot 10^{-10}\right)^2} \approx 5.03 \cdot 10^{-9}N$$

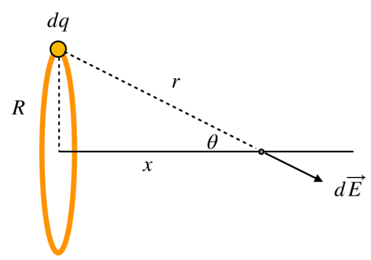
$$F_{456} = -\sqrt{3}k\frac{e^2}{(d\sqrt{2})^2} = -\sqrt{3} \cdot 8.99 \cdot 10^9 \frac{\left(1.602 \cdot 10^{-19}\right)^2}{\left(\sqrt{2} \cdot 2.82 \cdot 10^{-10}\right)^2} \approx -2.51 \cdot 10^{-9}N$$

$$F_7 = k\frac{e^2}{(d\sqrt{3}^2)} = 8.99 \cdot 10^9 \frac{\left(1.602 \cdot 10^{-19}\right)^2}{\left(\sqrt{3} \cdot 2.82 \cdot 10^{-10}\right)^2} \approx 0.97 \cdot 10^{-9}N$$

$$\therefore F_{total} = 3.49 \cdot 10^{-9}N$$

Problem 4

a) Derive the equation that gives the electric field on the axis of a charged ring. Where is the strength of this electric field maximum?



Denote q the charge of the ring, and take a small change of that charge dq, over a small part of the ring. Take a point in the axis of the ring, with distance x from the ring. Take x the axis of the ring.

Because of symmetry, the the fields in the plane perpendicular to the ring will cancel each other, and the only the components in the x axis will remain.

$$r = \sqrt{x^2 + R^2} \implies \mathrm{d}\vec{E} = k\frac{\mathrm{d}q}{x^2 + R^2}$$

$$\mathrm{d}\vec{E_x} = k\frac{\mathrm{d}q}{x^2 + R^2} \cdot \frac{x}{\sqrt{x^2 + R^2}}$$

Now we sum all those electric fields, to get the total electric field.

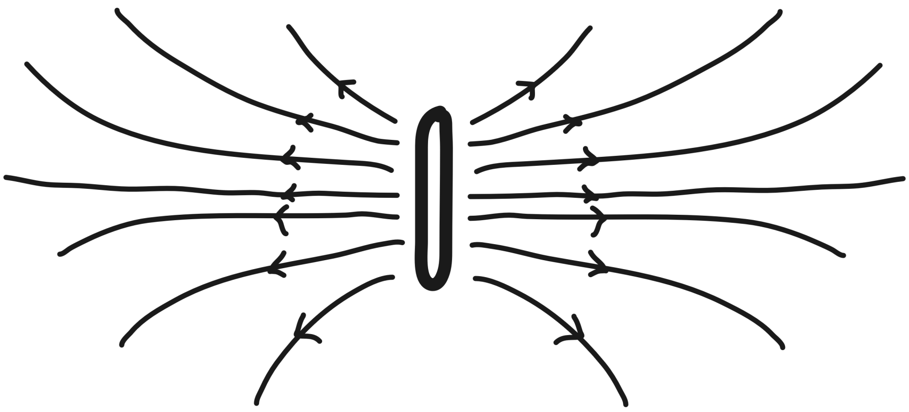
$$E = \int \frac{kx\,\mathrm{d}q}{(x^2 + R^2)^{\frac{3}{2}}} = \frac{kxq}{(x^2 + R^2)^{\frac{3}{2}}}$$

We can dfiferentiate, to find the maxmimum point.

$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{kq}{(x^2 + R^2)^{\frac{3}{2}}} - \frac{3kqx^2}{(x^2 + R^2)^{\frac{5}{2}}} = 0 \implies (x^2 + R^2) - 3x^2 = 0 \implies R^2 = 2x^2$$

$$\therefore x = R\frac{\sqrt{2}}{2}$$

b) Roughly sketch the electric field in the space surrounding the ring.



Problem 5

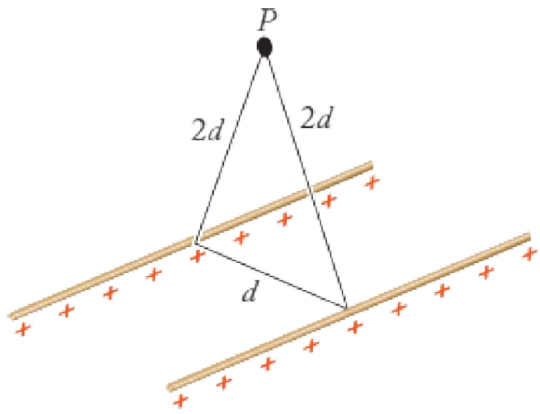
You wish to generate a uniform electric field of $2.0 \cdot 10^5 N/C$ in the space between two flat, parallel plates of metal placed face to face. The plates measure$0.30cm \times 0.30cm$. How much electric charge must you put on each plate? Assume that the gap between the plates is small so that the charge distribution and the electric field are approximately uniform, as for infinite plates.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \implies Q = EA\epsilon_0$$

$$Q = 2.0 \cdot 10^5 \cdot (0.003)^2 \cdot 8.85 \cdot 10^{-12} = \approx 1.59 \cdot 10^{-11} C$$

Problem 6

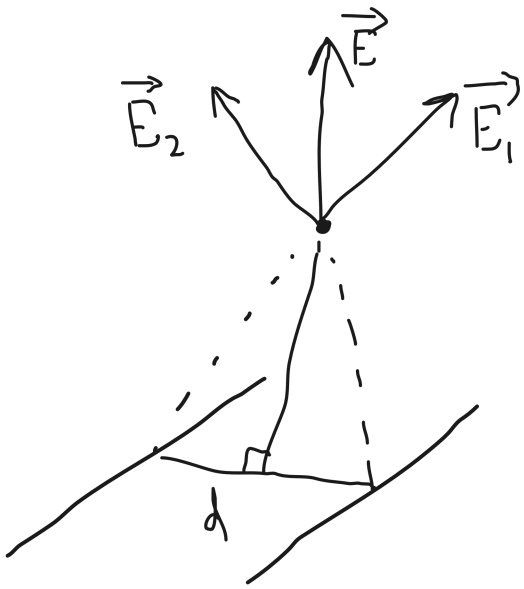
Each of two very long, straight, parallel rods carries a positive charge of λ coulombs per meter. The distance between the rods is d (Fig.). Find the electric field at a point equidistant from the rods, with a distance $2d$ from each rod. Draw a diagram showing the direction of the electric field.



$$E_1 = E_2 = \frac{2k\lambda}{2d} = \frac{k\lambda}{d} \text{ and } h = \frac{\sqrt{15}}{2}d$$

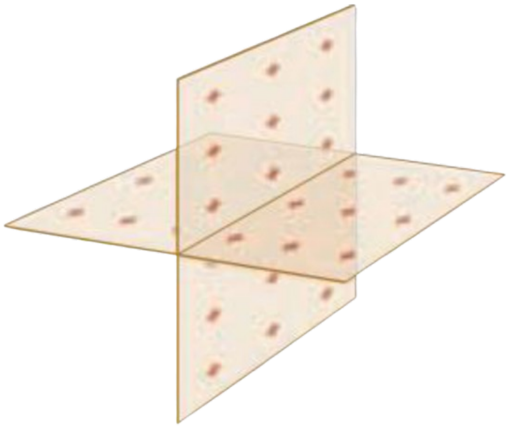
The horizontal components will cancel each other, and the cosine of the angle between the electric field and the vertical component is $\frac{\sqrt{15}}{4}$

$$\therefore E = 2E_1 \cos\theta = \frac{k\lambda\sqrt{15}}{2d}$$



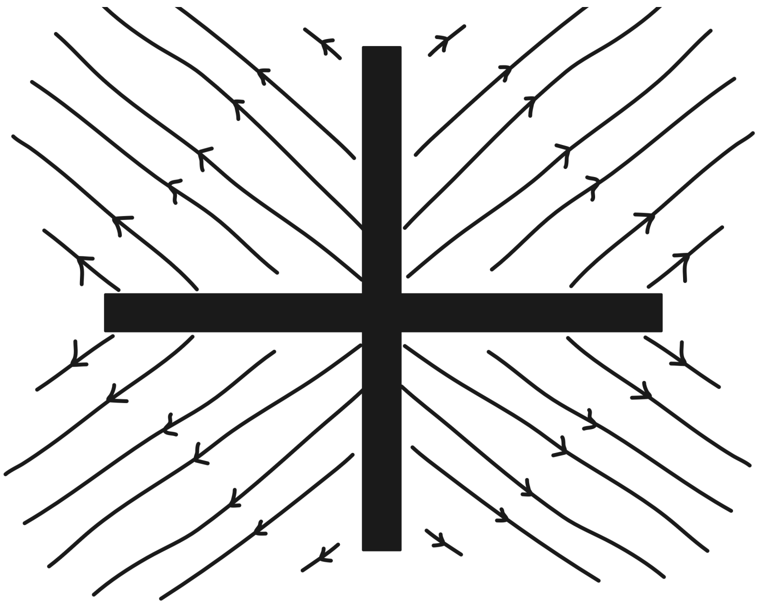
Problem 7

Two large sheets of paper intersect at right angles. Each sheet carries a uniform charge distribution of positive charge (Fig.). The charge per unit area on the sheets is $3.0 \cdot 10^{-6} C/m^2$. Find the magnitude of the electric field in each of the four quadrants. Sketch the field lines in each quadrant.



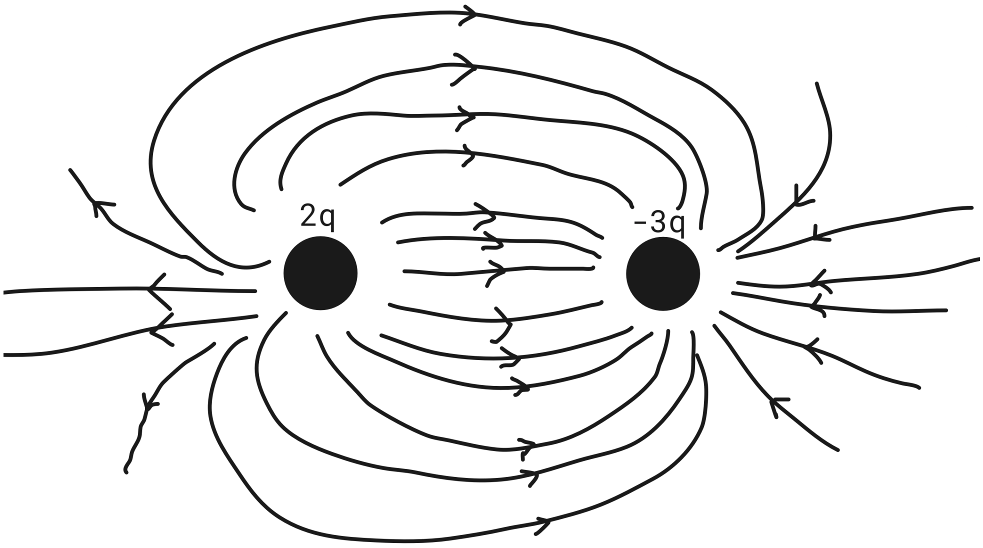
For a certain point, we will have two electric fields, one from the first sheet, and another, from the other sheet, and they will be perpendicular to each other. Moreover, since they have the same charge per unit area, the sum of those vectors will be $\sqrt{2}$ times the magnitude of one.

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0} \implies E = \frac{\sigma}{\sqrt{2}\varepsilon_0} = \frac{3.0 \cdot 10^{-6}}{\sqrt{2} \cdot 8.85 \cdot 10^{-12}} \approx 2.39 \cdot 10^5 N/C$$



Problem 8

A positive charge $2q$ and a negative charge $-3q$ are separated by a distance d. Sketch the field lines of the electric field produced jointly by these charges in a plane containing the two charges.



Problem 9

The dipole moment of an HCl molecule is $\mu = 3.4 \cdot 10^{-30} Cm$. Calculate the magnitude of the torque that an electric field of $2.0 \cdot 10^6 N/C$ exerts on this molecule when the angle between the electric field and the longitudinal axis of the molecule is $45°$.

Relative charge is the same as one electron, for both atoms in the dipole.

$$\mu = q \cdot r \implies r = \frac{\mu}{e} \text{ and } F_1 = F_2 = Ee$$

Assume the axis of rotation is the middle of the bond between the hydrogen and chlorine atoms.

$$\tau_1 = \frac{r}{2} F_1 \sin\theta = \frac{r}{2} F_2 \sin\theta = \tau_2 \implies \tau = \tau_1 + \tau_2 = rF_1 \sin\theta = \mu E \sin\theta$$

$$\therefore \tau = 3.4 \cdot 10^{-30} \cdot 2.0 \cdot 10^6 \cdot \frac{\sqrt{2}}{2} \approx 4.81 \cdot 10^{-24} Nm$$