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# ENGS123 Electricity and Magnetism - Homework 3

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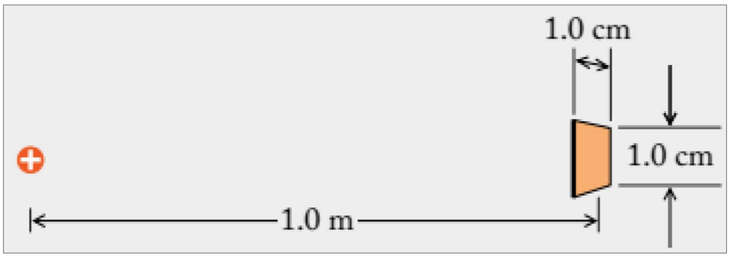
## Problem 1

A small square surface, $1.0cm \cdot 1.0cm$, is placed at a distance of $1.0m$ from a point charge of $3.0 \cdot 10^{-19}C$. What is the approximate electric flux through this square if it is face on to the electric field (see Fig.)? If it is tilted by 30°? If it is tilted by 60°?




$$E = k\frac{q}{r^2}, \Phi = ES\cos\varphi = k\frac{qa^2}{r^2}\cos\varphi$$

$$\Phi_0 = 8.99 \cdot 10^9 \cdot \frac{3.0 \cdot 10^{-19} \cdot (1.0 \cdot 10^{-2})^2}{1^2}\cos 0 \approx 2.70 \cdot 10^{-13}Nm^2/C$$

$$\Phi_1 = \Phi_0 \cos\frac{\pi}{6} \approx 2.34 \cdot 10^{-13}Nm^2/C \text{ and } \Phi_2 = \Phi_0 \cos\frac{\pi}{3} \approx 1.35 \cdot 10^{-13}Nm^2/C$$

## Problem 2

A uniform electric field is given by $\vec{E} = \left(3.0\frac{N}{C}\right)\hat{\imath} + \left(2.0\frac{N}{C}\right)\hat{\jmath} + \left(1.0\frac{N}{C}\right)\hat{k}$. What is the flux through a flat, $4.0m^2$ area that lies in the y–z plane? What if that same area instead has its surface normal along an octantdiagonal, so that the normal unit vector is $\hat{n} = \left(\frac{1}{\sqrt{3}}\right)\hat{\imath} + \left(\frac{1}{\sqrt{3}}\right)\hat{\jmath} + \left(\frac{1}{\sqrt{3}}\right)\hat{k}$?

Only the electric field lines of the $x$ axis are orthogonal to the flat area, and only it will create electric flux.

$$\Phi_1 = ES = 3.0 \cdot 4.0 = 12Nm^2/C$$

$$\Phi_2 = \vec{E} \cdot \vec{S} = \frac{12}{\sqrt{3}} + \frac{8}{\sqrt{3}} + \frac{4}{\sqrt{3}} = \frac{24}{\sqrt{3}} \approx 13.9Nm^2/C$$

## Problem 3

a) A point charge of $2.0 \cdot 10^{-12} C$ sits at some distance above the x–y plane. What is the electric flux that this charge generates through the (infinite) x–y plane? Does the electric flux depend on the distance?

Since the plane is infinite, exactly half of the charge's electric fields will go through the $x - y$ plane.

$$\Phi_0 = \frac{q}{\varepsilon_0} \implies \Phi_0 \text{ is the flux all around the point charge}$$

$$\Phi = \frac{q}{2\varepsilon_0} = \frac{2.0 \cdot 10^{-12}}{2 \cdot 8.85 \cdot 10^{-12}} \approx 0.113 Nm^2/C$$

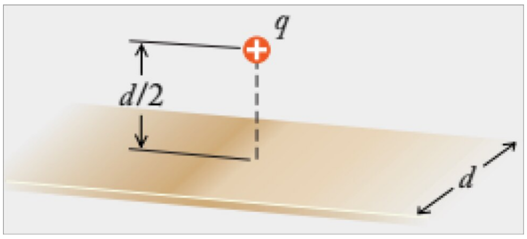
And this flux doesn't depend on the distance, because no matter the distance, half of the field lines will penetrate the $x - y$ plane.

b) A point charge of $2.0 \cdot 10^{-12} C$ is located at the center of a cubical Gaussian surface. What is the electric flux through each of the faces of the cube?

Since the charge is in the center and the cube is symmetric from the charge, the flux inside any of the faces will be a sixth of the total flux.

$$\Phi = \frac{q}{6\varepsilon_0} = \frac{2.0 \cdot 10^{-12}}{6 \cdot 8.85 \cdot 10^{-12}} \approx 3.77 \times 10^{-2} Nm^2/C$$

c) A point charge is placed at a distance $\frac{d}{2}$ from an infinitely long flat strip of width $d$ carrying a uniform charge distribution of $\sigma$ coulombs per $m^2$ (see Fig.). What is the electric flux that this point charge produces through the strip? (Hint: The strip is one of the four faces of a rectangular tube, or pipe, surrounding the point charge.)




As the hint suggests, imagine a long square tube with sides $d$. Our point charge will be coincidentally in the center of the square. The flux in our strip will be one fourth of the total flux produced by the point charge.

$$\Phi = \frac{q}{4\varepsilon_0}$$

## Problem 4

Prove that if the electric field is uniform in some region, then the charge density must be zero around any point in that region. (Hint: Use Gauss' Law.)

Assume the charge density is $\rho$, which is not zero around a point in a region with uniform a electric field.

$$\Phi = ES = \frac{Q_{\text{inside}}}{\varepsilon_0} \implies E = \frac{\rho V}{\varepsilon_0 4\pi r^2}$$

We disregard the outside field, because it has no effect in the inside. Since for any value of $r$, our electric field should be constant, the charge density shall be 0, so that there is no change in the electric field inside the region and only the outside electric field stays, which is uniform.

## Problem 5

A spherical rubber balloon has a uniform distribution of charge over its surface. Show that the electric field that this charge produces in the (empty) interior of the balloon is exactly zero.

Divide the sphere into small little spheres with the same center as the rubber balloon and use Gauss's law.

$$\Phi = ES = \frac{Q_{\text{inside}}}{\varepsilon_0}$$

For all those imaginary spheres, the charge inside the sphere will be zero, hence, the electric field will also be zero inside the rubber sphere.

## Problem 6

Charge is uniformly distributed over the volume of a very long cylindrical plastic rod of radius $R$. The amount of charge per meter of length of the rod is $\lambda$. Find a formula for the electric field at a distance $r$ from the axis of the rod. Assume $r < R$.

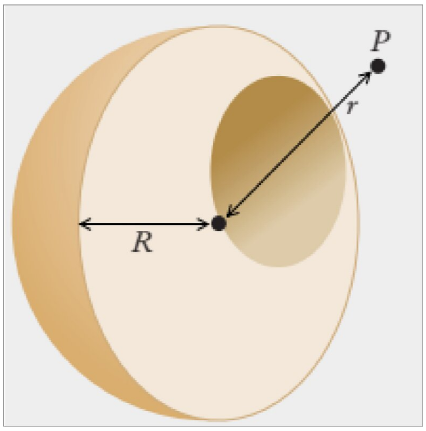
$$\Phi = ES = \frac{Q_{\text{inside}}}{\varepsilon_0}$$

Since the charges outside the radius $r$ will have no effect on the electric field, we can disregard them. Hence the charge of the inside will be $Q_{\text{inside}} = \frac{r^2}{R^2} Q_{\text{total}}$, by comparing the areas of the intersection circles.

$$E = \frac{Q_{\text{inside}}}{\varepsilon_0 S} = \frac{\lambda l r}{\varepsilon_0 2\pi R^2 l} = \frac{\lambda r}{\varepsilon_0 2\pi R^2}$$

## Problem 7

Positive charge $Q$ is uniformly distributed over the volume of a solid sphere of radius $R$. Suppose that a spherical cavity of radius $\frac{R}{2}$ is cut out of the solid sphere, the center of the cavity being at a distance of $\frac{R}{2}$ from the center of the original solid sphere (Fig.); the cut-out material and its charge are discarded. What new electric field does the sphere with the cavity produce at the point $P$ at a distance $r$ from the original center? Consider two cases




a) $r > R$

Assume the sphere is not hollow and calculate the electric field of it using the Gauss law.

$$E_0 = \frac{\rho \frac{4}{3}\pi R^3}{\varepsilon_0 4\pi r^2} = \frac{\rho R^3}{3\varepsilon r^2}$$

Now subtract the field created by the hollow part.

$$E_1 = \frac{\rho \frac{4}{3}\pi \left(\frac{R}{2}\right)^3}{\varepsilon_0 4\pi \left(r - \frac{R}{2}\right)^2} = \frac{\rho \left(\frac{R}{2}\right)^3}{3\varepsilon_0 \left(r - \frac{R}{2}\right)^2}$$

$$\therefore E = E_0 - E_1 = \frac{\rho R^3}{3\varepsilon r^2} - \frac{\rho \left(\frac{R}{2}\right)^3}{3\varepsilon_0 \left(r - \frac{R}{2}\right)^2}$$

b) $r < R$

Same thing, but the charges are different. Assume the sphere is not hollow and calculate the electric field of it using the Gauss law. The charges outside the sphere do not contribute to the electric field.

$$E_0 = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0 4\pi r^2} = \frac{\rho r}{3\varepsilon}$$

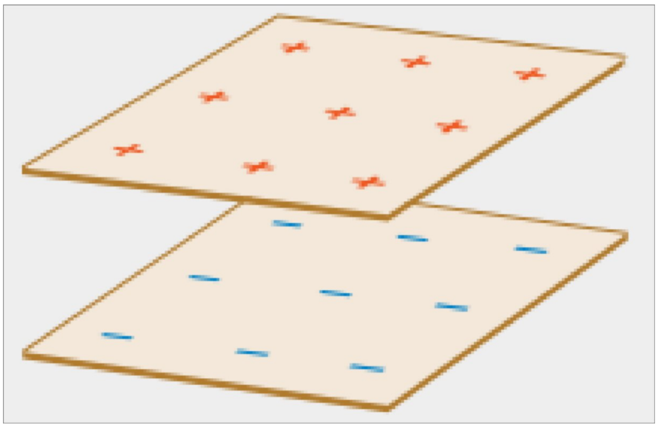
Now subtract the field created by the hollow part.

$$E_1 = \frac{\rho \frac{4}{3}\pi \left(r - \frac{R}{2}\right)^3}{\varepsilon_0 4\pi \left(r - \frac{R}{2}\right)^2} = \frac{\rho \left(r - \frac{R}{2}\right)}{3\varepsilon}e$$

$$\therefore E = E_0 - E_1 = \frac{\rho r}{3\varepsilon} - \frac{\rho \left(r - \frac{R}{2}\right)}{3\varepsilon} = \frac{\rho R}{6\varepsilon}$$

## Problem 8

Consider two large parallel metallic plates with uniform, opposite charge distributions, as in Fig(a). Suppose that the magnitude of the charge density on each plate is $2.0 \cdot 10^{-5} C/m^2$. The upper plate is positive and the lower negative. What is the magnitude of the electric field in the region between the plates?



We have a homemade capacitor. The formula for the electric field inside a capacitor was used.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{2.0 \cdot 10^{-5}}{8.85 \cdot 10^{-12}} \approx 2.26 \cdot 10^6 N/C$$

Derivation: Imagine a Gaussian cylinder around one of the plates.

$$\Phi = ES = \frac{Q_{\text{inside}}}{\varepsilon_0} \implies E = \frac{\sigma S_{\text{circle}}}{\varepsilon_0 2 S_{\text{circle}}} = \frac{\sigma}{2\varepsilon_0}$$

For two plates with opposite charges of the same magnitude, the fields would add up giving the formula for the electric field in a capacitor.