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ENGS123 Electricity and Magnetism - Homework 4

Mher Saribekyan A09210183

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Problem 1

The nucleus of lead is a uniformly charged sphere with a charge of 82e and a radius of $7.1 \cdot 10^{-15}m$. What is the electrostatic potential at the nuclear surface? At the nuclear center (bring derivation of the formula)?

Part 1: Find the electric field outside the sphere.

$$\Phi = ES = \frac{q}{\varepsilon_0} \implies E = \frac{q}{4\pi r^2 \varepsilon_0} = k\frac{q}{r^2}$$

$$V_1 = -\int_{\infty}^{R} E \,\mathrm{d}r = -\int_{\infty}^{R} k\frac{q}{r^2} \,\mathrm{d}r = kq\left[\frac{1}{r}\right]_{\infty}^{R} = \frac{kq}{R} = \frac{8.99 \cdot 10^9 \cdot 82 \cdot 1.602 \cdot 10^{-19}}{7.1 \cdot 10^{-15}} \approx 1.66 \cdot 10^7 V$$

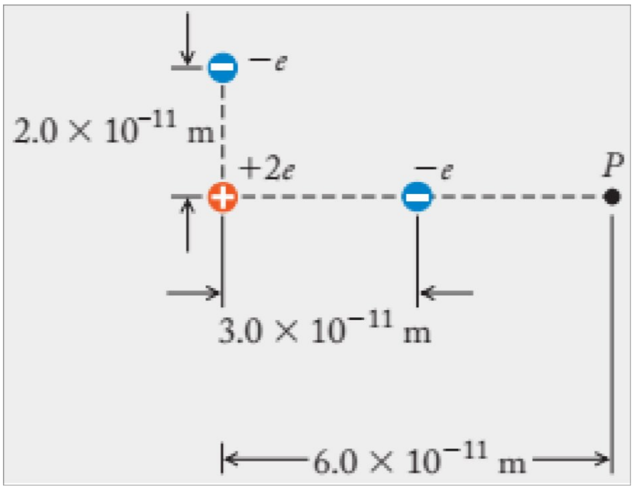
Part 2: Find the electric field inside the sphere, and integrate from the surface to the center.

$$\Phi = ES = \frac{\left(\frac{r}{R}\right)^3 q}{\varepsilon_0} \implies E = \frac{\left(\frac{r}{R}\right)^3 q}{4\pi r^2 \varepsilon_0} = k\frac{rq}{R^3}$$

$$V_0 = V_1 - \int_{R}^{0} E \,\mathrm{d}r = \frac{kq}{R} - \frac{kq}{R^3}\int_{R}^{0} r \,\mathrm{d}r = kq\left(\frac{1}{R} + \frac{1}{2R}\right) = \frac{3kq}{2R} = \frac{3}{2}V_1 \approx 2.49 \cdot 10^7 V$$

Problem 2

In a helium atom, at some instant one of the electrons is at a distance of $3.0 \cdot 10^{-11}m$ from the nucleus and the other electron is at a distance of $2.0 \cdot 10^{-11}m$, $90°$ away from the first. Find the electric potential produced jointly by the two electrons and the nucleus at a point P beyond the first electron and at a distance of $6.0 \cdot 10^{-11}m$ from the nucleus.



We need find the sum of the potentials created by the three different charges. We can clearly see that the potential created by the positive charge is fully canceled by the charged created by the first electron, at the point P.

$$\varphi = 8.99 \cdot 10^9 \left(\frac{2(1.602 \cdot 10^{-19})}{6.0 \cdot 10^{-11}} + \frac{-1.602 \cdot 10^{-19}}{3.0 \cdot 10^{-11}} + \frac{-1.602 \cdot 10^{-19}}{2\sqrt{10} \cdot 10^{-11}} \right) \approx -22.8V$$

Problem 3

An electron is initially a distance of $r_0 = 4.3 \cdot 10^{-9}$ from a proton, traveling directly away from the proton at speed $4.0 \cdot 10^5 m/s$. What is its speed when it is very far from the proton?

$$E_1 = E_{K1} + E_{P1} = \frac{m_e v_0^2}{2} - k\frac{e^2}{r} = \frac{m_e v_1^2}{2} = E_{K2} = E_2$$

$$v_1 = \sqrt{v_0^2 - \frac{2ke^2}{rm_e}} = \sqrt{(4.0 \cdot 10^5)^2 - \frac{2 \cdot 8.99 \cdot 10^9 \cdot (1.602 \cdot 10^{-19})^2}{4.3 \cdot 10^{-9} 9.109 \cdot 10^{-31}}} \approx 2.05 \cdot 10^5 m/s$$

Problem 4

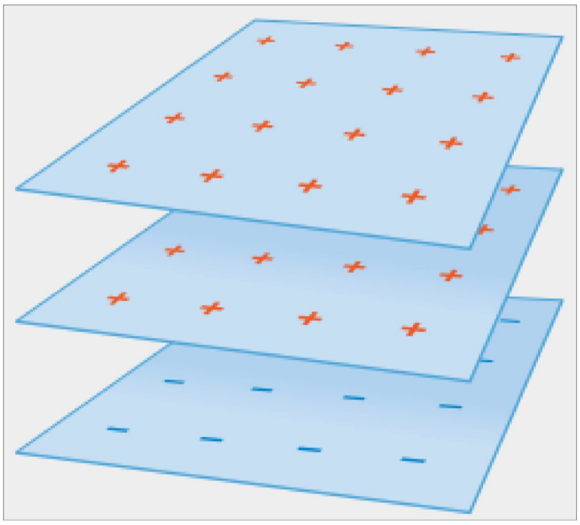
An alpha particle is initially at a very large distance from a plutonium nucleus. What is the minimum kinetic energy with which the alpha particle must be launched toward the nucleus if it is to make contact with the nuclear surface? The plutonium nucleus is a sphere of radius $7.5 \cdot 10^{-15} m$ with a charge of $94e$ uniformly distributed over the volume. For the purpose of this problem, the alpha particle may be regarded as a particle (of negligible radius) with a charge of $2e$. Express the answer in electronvolt units.

Calculate the energy required to bring the alpha particle from the very large distance to the surface of the nucleus:

$$E = \int_{\infty}^{R} -k\frac{qQ}{r^2}\,\mathrm{d}r = k\frac{qQ}{R} = 8.99 \cdot 10^9 \frac{2 \cdot (1.602 \cdot 10^{-19}) \cdot 94e}{7.5 \cdot 10^{-15}} \approx 3.61 \cdot 10^7 eV$$

Problem 5

Three large charged sheets are parallel to the x–z plane. The sheets are at $y = 0$, $y = d$, and $y = 2d$; these sheets have uniform surface charge densities $-\sigma$, $+2\sigma$, and $+\sigma$, respectively. Assuming the reference potential is zero at $y = 0$, determine the potential as a function of y.



The electric fields by created by those charged sheets are as follows:

$$E_1 = -\frac{\sigma}{2\varepsilon_0}, E_2 = \frac{\sigma}{\varepsilon_0}, E_3 = \frac{\sigma}{2\varepsilon_0}$$

$$V_1 = \frac{|y|\sigma}{2\varepsilon_0} + c_1, V_2 = -\frac{|y-d|\sigma}{\varepsilon_0} + c_2, V_3 = -\frac{|y-2d|\sigma}{2\varepsilon_0} + c_3, c_1, c_2, c_3 \in \mathbb{R}$$

$$\therefore V = \frac{|y|\sigma}{2\varepsilon_0} - \frac{|y-d|\sigma}{\varepsilon_0} - \frac{|y-2d|\sigma}{2\varepsilon_0} + c, c \in \mathbb{R}$$

a) Find the strength of the electric field E above the sheets, below the sheets, and in the spaces between the sheets. Find the direction of E at each place.

   We have four spaces, for simplicity reason, number them from A to D, A being the space bellow the plates.

$$E_A = -E_1 - E_2 - E_3 = -\frac{\sigma}{\varepsilon_0}$$

$$E_B = E_1 - E_2 - E_3 = -\frac{2\sigma}{\varepsilon_0}$$

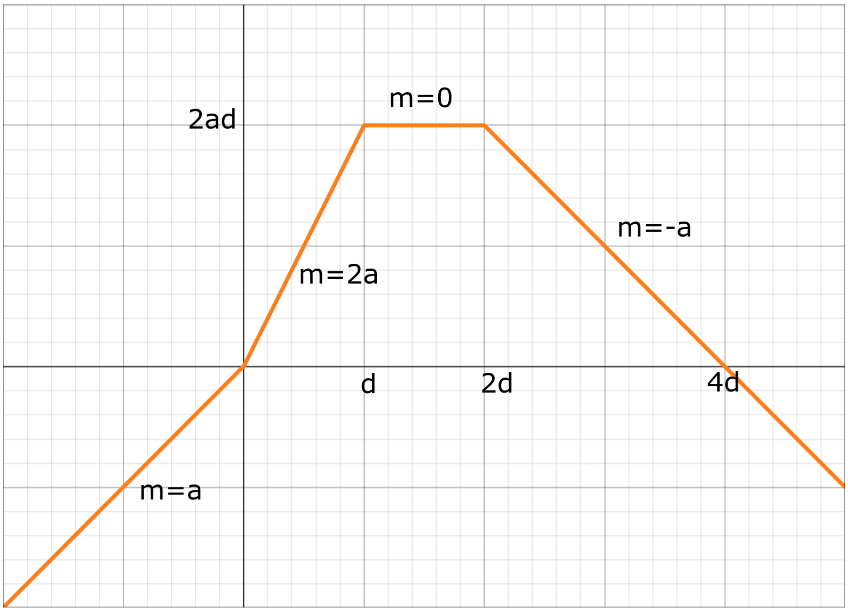
$$E_C = E_1 + E_2 - E_3 = 0$$

$$E_D = E_1 + E_2 + E_3 = \frac{\sigma}{\varepsilon_0}$$

   The direction of each E can be determined by its sign: positive shows the $+y$ direction.

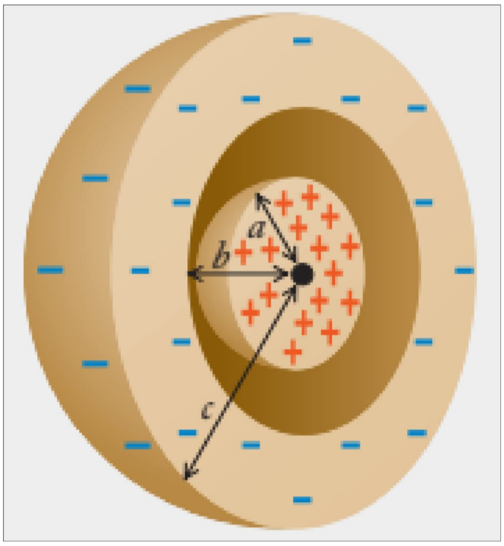
b) Draw the graph of potential dependence on the distance from the lower plate. Mention where you have taken the potential as zero. Draw the graph of potential vs y.

   The zero potential was taken at the region where the electric field is zero. Take $a = \dfrac{\sigma}{\varepsilon_0}$



Problem 6

A uniformly charged sphere of radius a is surrounded by a uniformly charged concentric spherical shell of inner radius b and outer radius c. The total charge in the sphere is Q, and that on the outer shell $-Q$. Find the potential at $r = c$, at $r = b$.



$$V_c = -\int_{\infty}^{c} k\frac{-Q}{r^2}\,\mathrm{d}r - \int_{\infty}^{c} k\frac{Q}{r^2}\,\mathrm{d}r = 0$$

To calculate the potential at b, we need to find the potential inside the larger sphere. For that we need to find the potential going from c to b.

$$E = -k\frac{rQ}{(c-b)^3} \implies V_1 = \int_{c}^{b} k\frac{rQ}{(c-b)^3}\,\mathrm{d}r = -\frac{kQ(c^2-b^2)}{2(c-b)^3}$$

$$V_2 = \frac{kQ}{b} \implies V_b = \frac{kQ}{b} - \frac{kQ(c^2-b^2)}{2(c-b)^3}$$

Problem 7

In some region of space, the electrostatic potential is the following function of x and y, but not of z:

$$V = x^2 + 2xy$$

where the potential is measured in volts and the distance in meters. Find the electric field at the point $x = 2m$, $y = 2m$.

$$\vec{E} = -\nabla\vec{V} = \left(-\frac{\partial\vec{V}}{\partial x}, -\frac{\partial\vec{V}}{\partial y}, -\frac{\partial\vec{V}}{\partial z}\right) = (-8, -4, 0)$$

$$\therefore E = 4\sqrt{5}N/C$$