Digital version

ENGS123 Electricity and Magnetism - Homework 6

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Problem 1

The collector of an electrostatic machine is a metal sphere of radius $18cm$.

a) What is the capacitance of this sphere?

$$C = \frac{Q}{V} = \frac{Q}{k\frac{Q}{R}} = \frac{R}{k} = \frac{0.18}{8.99 \cdot 10^9} \approx 2.0 \cdot 10^{-11} F$$

b) How many coulombs of charge must you place on this sphere to raise its potential to $2.0 \cdot 10^5 V$?

$$Q = CV \approx 2.0 \cdot 10^{-11} \cdot 2.0 \cdot 10^5 = 4.0 \cdot 10^{-6} C$$

Problem 2

A pair of parallel conducting plates, each of size $30cm \times 30cm$, are separated by a gap of $1.0mm$

a) How much work must you do against the electric forces to charge these plates with $+1.0 \cdot 10^{-6}C$ and $-1.0 \cdot 10^{-6}C$, respectively?

$$E = U_0 = \frac{Q^2}{2\frac{A\varepsilon_0}{d}} = \frac{(1.0 \cdot 10^{-6})^2}{2\frac{(30 \cdot 10^{-2})^2 \cdot 8.85 \cdot 10^{-12}}{1 \cdot 10^{-3}}} \approx 6.28 \cdot 10^8 J$$

b) How much work must you do against the electric forces to increase the distance between plates up to $2mm$ (plates are disconnected from voltage source)?

$$U_1 = \frac{Q^2}{2\frac{A\varepsilon_0}{d}} = \frac{(1.0 \cdot 10^{-6})^2}{2\frac{(30 \cdot 10^{-2})^2 \cdot 8.85 \cdot 10^{-12}}{2 \cdot 10^{-3}}} \approx 1.26 \cdot 10^9 J \implies E = U_1 - U_0 = 6.28 \cdot 10^8 J$$

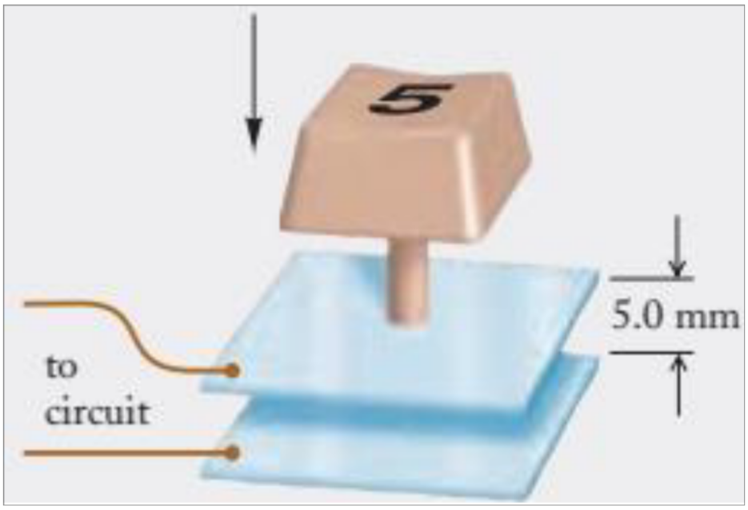
c) How much work must you do against the electric forces to increase the distance between plates up to $2mm$ (plates are connected to a voltage source providing $5V$ of potential difference between plates)?

$$U_2 = \frac{CV^2}{2} = \frac{\frac{(30 \cdot 10^{-2})^2 \cdot 8.85 \cdot 10^{-12}}{1 \cdot 10^{-3}} \cdot 5^2}{2} \approx 9.96 \cdot 10^{-9} J$$

$$U_3 = \frac{U_2}{2} \implies U = U_3 - U_2 = 4.98 \cdot 10^{-9} J$$

Problem 3

In many computer keyboards the switches under the keys consist of small parallel-plate capacitors (see Fig.). The key is attached to the upper plate, which is movable. When you push the key down, you push the upper plate toward the lower plate, and you alter the plate separation d and the capacitance. The capacitor is connected to an external circuit that maintains a constant potential difference ΔV across the plates. The change of capacitance therefore sends a pulse of charge from the capacitor into the computer circuit. Suppose that the initial plate separation is $5.0mm$ and the initial capacitance is $6.0 \cdot 10^{-13}F$. The final plate separation (with the key fully depressed) is $0.20mm$. The constant potential difference is $8.0V$. What is the change in capacitance when you depress the key? What is the amount of electric charge that flows out of the capacitor into the computer circuit?

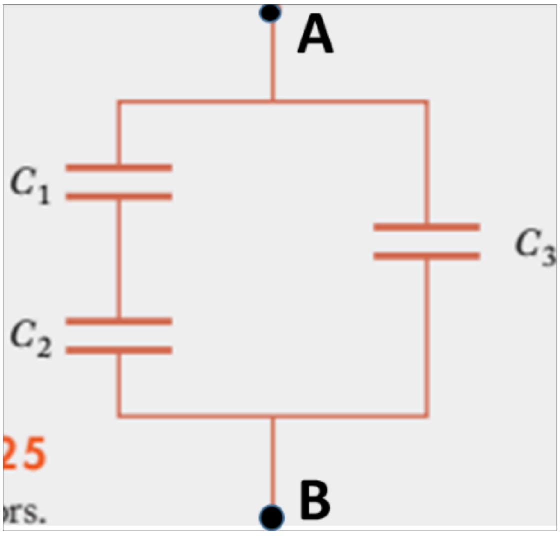


$$C_0 = \frac{\varepsilon_0 A}{d_0} \implies C_1 = \frac{\varepsilon_0 A}{d_1} = \frac{d_0}{d_1} C_0 = \frac{5}{0.2} \cdot 6.0 \cdot 10^{-13} = 1.5 \cdot 10^{-11} F$$

$$\Delta Q = Q_0 - Q_1 = V(C_1 - C_0) = 8(1.5 \cdot 10^{-11} - 6.0 \cdot 10^{-13}) \approx 1.15 \cdot 10^{-10} C$$

Problem 4

Three capacitors with capacitances $C_1 = 5.0\mu F$, $C_2 = 3.0\mu F$, and $C_3 = 8.0\mu F$ are connected as shown in Fig. computer circuit?



a) Find their combined capacitance.

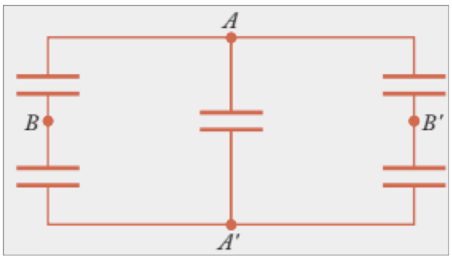
$$C = \frac{5.0 \cdot 3.0}{5.0 + 3.0} \mu F + 8.0 \mu F = 9.875 \mu F$$

b) The voltage between A and B terminals is $12V$. Find the voltage across each of the capacitors.

$$Q_1 = Q_2 \implies \begin{cases} C_1 = \dfrac{Q}{V_1} \\ C_2 = \dfrac{Q}{12 - V_1} \end{cases} \implies (12 - V_1)3 = 5V_1 \implies V_1 = \frac{36}{8} = 4.5V, V_2 = 7.5V, V_3 = 12V$$

Problem 5

Figure shows five capacitors of $4.0F$ each connected together. Find the net capacitance of this combination between the terminals:



a) A and A'

$$C = 2 + 4 + 2 = 8.0F$$

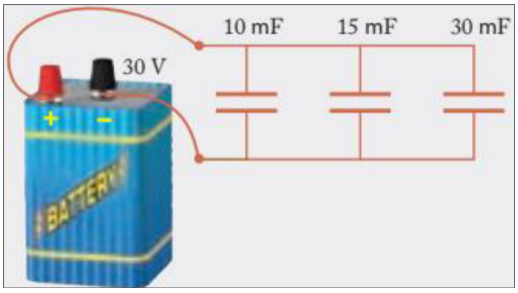
b) B and B'

Since all the capacitors have the same values, the voltage across A and A' will be zero, thus the middle capacitor can be disregarded as no charge will be stored on it.

$$C = 2 + 2 = 4.0F$$

Problem 6

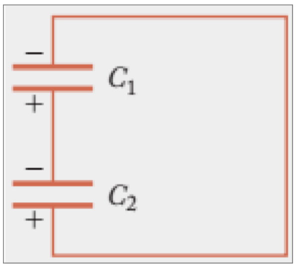
What is the total charge stored on the three capacitors connected to a 30-V battery as shown in Figure? What is the total energy stored in the three capacitors?



$$Q_{\text{total}} = Q_1 + Q_2 + Q_3 = 30(10 + 15 + 30) \cdot 10^{-3} = 1.65C$$
$$E = \frac{Q^2}{2C} = \frac{1.65^2}{2 \cdot 55 \cdot 10^{-3}} = 24.75J$$

Problem 7

Two capacitors, of $2.0F$ and $6.0F$, respectively, are initially charged to $24V$ by connecting each, for a few instants, to a $24V$ battery. The battery is then removed and the charged capacitors are connected in a closed series circuit, the positive terminal of each capacitor being connected to the negative terminal of the other Figure. What is the final charge on each capacitor? What is the final potential difference across each?



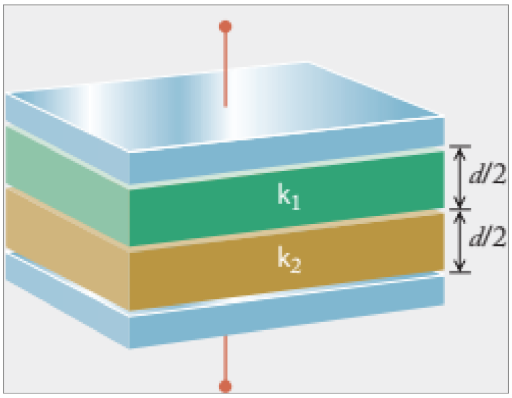
We basically create a short circuit. When connected, the first capacitor will cancel its charge and be charged with the opposite polarity, as it initially had less charge than the second capacitor. In the circuit we have a parallel connection, meaning the voltages across both capacitors should be equal

$$Q_{\text{total}} = 24(6 - 2) = 96C \text{ and } V_1 = V_2 \implies C_1V + C_2V = 96 \implies V_1 = V_2 = 12V$$

$$\therefore Q_1 = 24C, Q_2 = 72C$$

Problem 8

A parallel-plate capacitor of plate area A and spacing d is filled with two parallel slabs of dielectric of equal thickness with dielectric constants k_1 and k_2, respectively (Fig.). What is the capacitance? (Hint: Check that the configuration of Fig. is equivalent to two capacitors in series.)

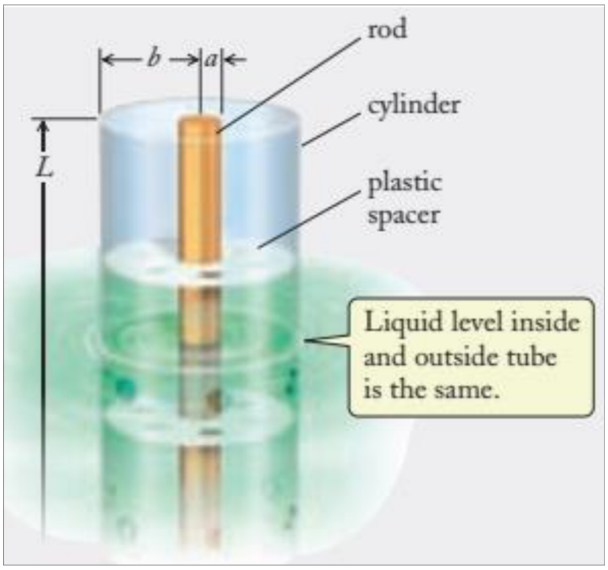


The voltage at touching point of both dielectrics has the same voltage potential, therefore we can assume the capacitor acts as two capacitors in series.

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{\varepsilon_1 \varepsilon_0 A}{\frac{d}{2}} \frac{\varepsilon_2 \varepsilon_0 A}{\frac{d}{2}}}{\frac{\varepsilon_1 \varepsilon_0 A}{\frac{d}{2}} + \frac{\varepsilon_2 \varepsilon_0 A}{\frac{d}{2}}} = \frac{2\varepsilon_1 \varepsilon_2 \varepsilon_0 A}{d(\varepsilon_1 + \varepsilon_2))}$$

Problem 9

A sensor for measuring liquid levels is made from a cylindrical capacitor (Fig) of length $L = 50cm$. The inner conductor has radius $a = 1.0mm$ and the outer conducting shell has radius $b = 4.0mm$. If the sensor is used to detect the level of liquid nitrogen ($k = 1.433$), what is its capacitance when it is:



a) empty,

$$C_0 = 2\pi\varepsilon_0 \frac{L}{\ln\left(\frac{b}{a}\right)} = 2\pi 8.85 \cdot 10^{-12} \frac{0.50}{\ln\left(\frac{4.0}{1.0}\right)} \approx 2.01 \cdot 10^{-11} F$$

b) full?

$$C_1 = \varepsilon_1 C_0 \approx 1.433 \cdot 2.01 \cdot 10^{-11} \approx 2.88 \cdot 10^{-11} F$$