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ENGS123 Electricity and Magnetism - Homework 7

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October 17, 2024

Problem 1

A current begins to flow at $t = 0$ and increases with time according to $I(t) = At + Bt^2$, where $A = 0.50C/s^2$ and $B = 0.20C/s^3$. What is the current at $t = 5.0s$? What total charge has flowed by $t = 5.0s$?

$$I(5.0) = 0.50 \cdot 5 + 0.20 \cdot 5^2 = 7.5A$$

$$Q = \int_0^5 I(t)\,\mathrm{d}t = \frac{0.50 \cdot 5^2}{2} + \frac{0.20 \cdot 5^3}{3} = \frac{175}{12} \approx 14.6C$$

Problem 2

When the starter motor of an automobile is in operation, the cable connecting it to the battery carries a current of $80A$. This cable is made of copper and is $0.50cm$ in diameter. What is the electric field in the cable?

$$E = \frac{I\rho}{S} = \frac{80 \cdot 1.7 \cdot 10^{-8}}{\pi \cdot (0.0025)^2} \approx 6.9 \cdot 10^{-2} N/m$$

Problem 3

The alternative form of Ohm's Law is $j = \sigma E$, where σ is the **electrical conductivity** and j current density (current per unit area of cross section). What is the electrical conductivity of copper?

$$\frac{j}{\sigma} = \frac{I\rho}{S} \implies \sigma = \frac{jS}{I\rho} = \frac{1}{\rho} \approx 5.88 \cdot 10^7 S/m$$

Problem 4

Although aluminum has a somewhat higher resistivity than copper, it has the advantage of having a considerably lower density. Find the mass of a $100m$ segment of aluminum cable $3.0cm$ in diameter. Compare this with that of a copper cable of the same length and the same resistance. The densities of aluminum and of copper are $2.7 \cdot 10^3 kg/m^3$ and $8.9 \cdot 10^3 kg/m^3$, respectively.

$$m_{Al} = \rho V = 100 \cdot \pi (0.015)^2 \cdot 2.7 \cdot 10^3 \approx 191kg$$

$$\rho_1 \frac{l}{S_1} = \rho_2 \frac{l}{S_2} \implies S_2 = \frac{\rho_2}{\rho_1} S_1$$

$$m_{Cu} = \rho V = 100 \cdot \frac{1.7 \cdot 10^{-8}}{2.8 \cdot 10^{-8}} \cdot \pi (0.015)^2 \cdot 8.9 \cdot 10^3 \approx 382kg$$

Problem 5

According to safety standards set by the American Boat and Yacht Council, the potential drop along a copper wire connecting a $12V$ battery to an item of electrical equipment should not exceed 10%, that is, it should not exceed $1.2V$. Suppose that a $9.0m$ wire (length measured around the circuit) carries a current of $25A$. What gauge of wire is required for compliance with the above standard? Use the table of wire gauges. Repeat the calculation for currents of $35A$ and $45A$.

GAUGE NO.	DIAMETER
8	0.3264 cm
9	0.2906
10	0.2588
11	0.2305
12	0.2053

$$R = \rho\frac{l}{S} = \frac{V}{I} \implies S_{\min} = \frac{\rho l I}{V} \implies d_{\min} = 2\sqrt{\frac{\rho l I}{\pi V}}$$

$$d_1 \geq 2\sqrt{\frac{1.7 \cdot 10^{-8} \cdot 9 \cdot 25}{\pi \cdot 1.2}} \approx 0.20cm \implies 12 \text{ Gauge}$$

$$d_2 \geq 2\sqrt{\frac{1.7 \cdot 10^{-8} \cdot 9 \cdot 35}{\pi \cdot 1.2}} \approx 0.24cm \implies 10 \text{ Gauge}$$

$$d_3 \geq 2\sqrt{\frac{1.7 \cdot 10^{-8} \cdot 9 \cdot 45}{\pi \cdot 1.2}} \approx 0.27cm \implies 9 \text{ Gauge}$$

Problem 6

The windings of high-current electromagnets are often made of copper pipe. The current flows in the walls of the pipe, and cooling water flows in the interior of the pipe. Suppose the copper pipe has an outside diameter of $1.20cm$ and an inside diameter of $0.80cm$. What is the resistance of $30m$ of this copper pipe? What voltage must be applied to it if the current is to be $600A$?

$$R = \rho\frac{l}{S} = 1.7 \cdot 10^{-8}\frac{30}{\pi((0.006)^2 - (0.004)^2)} \approx 8.1 \cdot 10^{-3}\Omega$$

$$V = IR \approx 600 \cdot 8.1 \cdot 10^{-3} \approx 4.87V$$

Problem 7

In the Van de Graaf generator a rezine tape of wide 30 cm is used. The tape moves at a speed of $20m/s$. The charge given to the tape at the bottom spinner produces an electric field of $E = 1.2 \cdot 10^6 V/m$. What is the current provided by the tape.

$$I = \frac{\Delta Q}{\Delta t} = \frac{\sigma S}{\Delta t} = \sigma v w = E\varepsilon_r\varepsilon_0 v w = 1.2 \cdot 10^6 \cdot 7 \cdot 8.85 \cdot 10^{-12} \cdot 20 \cdot 0.30 \approx 0.46mA$$