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# ENGS123 Electricity and Magnetism - Homework 8

Mher Saribekyan A09210183

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## Problem 1

During operation, the effective resistances of a toaster, a microwave oven, and an electric frying pan are $11\Omega$, $16\Omega$, and $12\Omega$, respectively. If the three are all plugged into a 115-volt outlet, what is the total current?

$$I = \frac{V}{R} = \frac{115}{11} + \frac{115}{16} + \frac{115}{12} \approx 27.2A$$

## Problem 2

An underground telephone cable, consisting of a pair of wires, has suffered a short somewhere along its length (Fig.). The telephone cable is $5.0km$ long, and in order to discover where the short is, a technician first measures the resistance across terminals AB; then he measures the resistance across terminals CD. The first measurement yields $30\Omega$; the second, $70\Omega$. Where is the short?

$$R_{\text{total}} = \frac{30+70}{2} = 50\Omega \implies S = \frac{15}{50} \cdot 5 = 1.5km$$

## Problem 3

A long, thin wire of resistance $R$ is cut into eight pieces. Four of these pieces are then placed side by side to form a new wire 1/8 of the original length. What is the resistance of the new wire?

$$R = \rho\frac{l}{S} \implies R_1 = R\frac{\frac{1}{8}}{4} = \frac{R}{32}$$

## Problem 4

Three resistors with $R_1 = 2.0\Omega$, $R_2 = 4.0\Omega$, and $R_3 = 6.0\Omega$ are connected as shown in Fig.

a) Find the net resistance of the combination.

$$R = 2 + \frac{4 \cdot 6}{4+6} = 4.4\Omega$$

b) Find the current that passes through the combination if a potential difference of $8.0V$ is applied to the terminals.

$$I = \frac{V}{R} = \frac{8}{4.4} \approx 1.82A$$

c) Find the potential drop and the current for each individual resistor.

$$I_1 \approx 1.82A \implies V_1 = I_1 R_1 \approx 3.64V \implies V_2 = V_3 \approx 4.36V \implies I_2 = \frac{V_2}{R_2} \approx 1.09A, I_3 \approx 0.73A$$

## Problem 5

Three resistors with $R_1 = 4.0\Omega$, $R_2 = 6.0\Omega$, and $R_3 = 8.0\Omega$ are connected as shown in Fig.

a) Find the net resistance of the combination.

$$R = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} = \frac{4 \cdot 6 \cdot 8}{4 \cdot 6 + 6 \cdot 8 + 4 \cdot 8} \approx 1.85\Omega$$

b) Find the current that passes through the combination if a potential difference of $12.0V$ is applied to the terminals.

$$I = \frac{V}{R} \approx \frac{12}{1.85} = 6.5A$$

c) Find the potential drop and the current for each individual resistor.

$$V_1 = V_2 = V_3 = 12.0V, I_1 = 3A, I_2 = 2A, I_3 = 1.5A$$

## Problem 6

Five resistors with $R = 6.0\Omega$ are connected as shown in Fig. The power source provides $\mathcal{E} = 24V$ of EMF

a) Find the net resistance of the combination.

We have multiple combination of resistors, which can be calculated going from right to left, substituting the resulting combination with one resistor. Number the resistors from 1 to 5 going from left to right.

$$R_{45} = 2R, R_{345} = \frac{2}{3}R, R_{2345} = \frac{5}{3}R, R_{12345} = \frac{5}{8}R = 3.75\Omega$$

b) Find the current that passes through the combination.

$$I = \frac{V}{R} = \frac{24}{3.75} = 6.4A$$

c) Find the potential drop and the current for each individual resistor.

$$V_1 = 24V \implies I_1 = 4A$$

$$I_2 = I_{\text{total}} - I_1 = 2.4A \implies V_2 = 14.4V \implies V_3 = 9.6V \implies I_3 = 1.6A$$

$$I_4 = I_5 = I_2 - I_3 = 0.8A \implies V_4 = V_5 = 4.8V$$

## Problem 7

A circuit consists of a resistor of $3.000\Omega$ connected to a (resistanceless) battery. To measure the current in this circuit, you insert an ammeter of internal resistance $2.0 \cdot 10^{-3}\Omega$. This ammeter then reads $3.955A$. What was the current in the circuit before you inserted the ammeter?

$$R_{\text{total}} = 3.002\Omega \implies V \approx 11.99V \implies I = 3.998A$$

## Problem 8

The temperature coefficients of resistance of certain alloys are positive whereas others are negative. This makes it possible to combine wires of different alloys to construct a resistor that has a resisance that does not vary with temperature. Wires of constantan and manganin are available having resistances per unit length $r$ (measured at $0°C$ ) given in the table below. The temperature coefficients of resistance $\alpha$ are also listed. What lengths $L_c$ and $L_m$ of the constantan and manganin wires should be connected in series to form a resistor of constant resistance $5.0\Omega$?

$$L_m r_m (1 + \alpha_m \Delta T) + L_c r_c (1 + \alpha_c \Delta T) = 5$$

$$L_m r_m + L_c r_c + \Delta T (L_c r_c \alpha_c + L_m r_m \alpha_m) = 5$$

$$\begin{cases} L_c r_c + L_m r_m = 5 \\ L_c r_c \alpha_c + L_m r_m \alpha_m = 0 \end{cases} \implies \begin{cases} 5.3 L_m + 6.3 L_c = 5 \\ 1.4 \cdot 5.3 L_m = 3.0 \cdot 6.3 L_c \end{cases} \implies \begin{cases} L_m \approx 0.64m \\ L_c \approx 0.25m \end{cases}$$