Digital version

ENGS123 Electricity and Magnetism - Homework 9

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Problem 1

A voltmeter of internal resistance $5.0 \cdot 10^4 \Omega$ is connected across the poles of a $12 - V$ battery of internal resistance 0.020Ω.

a) What is the current flowing through the battery?

$$I = \frac{V}{R_1 + R_2} = \frac{12}{50000.020} \approx 2.4 \cdot 10^{-4} A$$

b) What is the voltage drop across the internal resistance of the battery?

$$V_1 = \frac{V R_1}{R_1 + R_2} \approx 4.8 \cdot 10^{-6} V$$

Problem 2

In an effort to recharge a "dead" automobile battery, a strong battery with EMF $\mathcal{E}_1 = 12.6V$ is connected to a weak battery with $\mathcal{E}_2 = 11.1V$, forcing charge backward through the weak battery, as in Fig. (note that the positive terminals of the two batteries are connected, as are the negative terminals). Assume that the resistance of the wires, resistors and the internal resistance of the batteries total 0.12Ω, and that the EMFs remain constant. How much current flows? How much energy is delivered to the weak battery in the first 30 seconds? Suppose that, by mistake, the positive terminal of each battery is connected to the negative terminal of the other, a dangerous arrangement. What current flows?

$$I_1 = \frac{12.6 - 11.1}{0.12} = 12.5A$$

$$A = VIt = (12.6 - 11.1) \cdot 12.5 \cdot 30 = 562.5J$$

$$I_2 = \frac{12.6 + 11.1}{0.12} = 197.5A$$ 💀

Problem 3

Consider the circuit shown in Fig. Given that $\mathcal{E}_1 = 5.0V$, $\mathcal{E}_2 = 3.0V$, $R_1 = 4.0\Omega$, $R_2 = 3.0\Omega$ and $R_3 = 2.0\Omega$, what is the current in each source? (Hint: Use the symmetry.). What is the current in each resistor? What is the potential difference between points P and P'?

$$\begin{cases} I_1R_1 + I_3R_3 = \mathcal{E}_1 \\ I_2R_2 + I_3R_3 = \mathcal{E}_2 \\ I_1 + I_2 = I_3 \end{cases} \implies \begin{cases} 4I_1 + 2I_3 = 5 \\ 3I_2 + 2I_3 = 3 \\ I_1 + I_2 = I_3 \end{cases} \implies \begin{cases} 30I_1 + 10I_2 = 25 \\ 4I_1 + 10I_2 = 6 \\ I_1 + I_2 = I_3 \end{cases} \implies \begin{cases} I_1 = I_{\mathcal{E}_1} = \frac{19}{26}A \approx 0.73A \\ I_2 = I_{\mathcal{E}_2} = \frac{8}{26}A \approx 0.31A \\ I_3 = \frac{27}{26}A \approx 1.04A \end{cases}$$

$$V_{PP'} = \mathcal{E}_1 - \mathcal{E}_2 = 2V$$

Problem 4

A battery of EMF $\mathcal{E}$ and internal resistance R_i is connected to an external circuit of resistance R. In terms of $\mathcal{E}$, R_i, and R, what is the power delivered by the battery to the external circuit? Show that this power is maximum if $R = R_i$.

$$I = \frac{\mathcal{E}}{R + R_i}, P = I^2R = \frac{\mathcal{E}^2R}{(R + R_i)^2}$$

$$P'_R = \frac{\mathcal{E}^2}{(R + R_i)^2} - \frac{2\mathcal{E}^2R}{(R + R_i)^3} = 0 \implies R + R_i = 2R \implies R = R_i$$

Problem 5

For the circuit shown in Fig., the switch S has been in position 2 for a long time. At t=0, it is moved to position 1.

a) Write an expression for the charge Q on the capacitor as a function of the time t.

$$V = \frac{Q}{C}, \frac{\mathrm{d}Q}{\mathrm{d}t}(R_1 + R_3) + \frac{Q}{C} = \mathcal{E} \implies \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathcal{E}C - Q}{(R_1 + R_3)C} \implies \int \frac{\mathrm{d}Q}{\mathcal{E}C - Q} = \int \frac{\mathrm{d}t}{(R_1 + R_3)C}$$

$$-\ln(\mathcal{E}C - Q) = \frac{t}{(R_1 + R_3)C} + c_1 \implies Q(t) = \mathcal{E}C - ce^{-\frac{t}{(R_1+R_3)C}}$$

$$Q(0) = \mathcal{E}C - c = 0 \implies Q(t) = \mathcal{E}C\left(1 - e^{-\frac{t}{(R_1+R_3)C}}\right)$$

b) Determine the voltage across resistor R_1 as a function of the time t.

$$I(t) = Q'(t) = \frac{-\mathcal{E}C}{(R_1 + R_3)C}\left(1 - e^{-\frac{t}{(R_1+R_3)C}}\right)$$

$$V_{R_1}(t) = I(t)R_1 = \frac{-\mathcal{E}CR_1}{(R_1 + R_3)C}\left(1 - e^{-\frac{t}{(R_1+R_3)C}}\right)$$

Problem 6

A voltmeter reads $11.9V$ when connected across the poles of a battery. The internal resistance of the battery is 0.020Ω. What must be the minimum value of the internal resistance of the voltmeter if the reading of the instrument is to coincide with the EMF of the battery to within better than 1.0%?

$$\mathcal{E} - 11.9 \leq 0.01\mathcal{E} \implies \mathcal{E} \leq \frac{11.9}{0.99}$$

$$\frac{\mathcal{E}R}{R + 0.020} = 11.9 \implies \mathcal{E} = \frac{11.9(R + 0.020)}{R} \leq \frac{11.9}{0.99} \implies R \geq \frac{0.99 \cdot 0.020}{0.01} = 1.98\Omega$$

Problem 7

Two batteries with internal resistances are connected in the circuit shown in Fig. Given that $R_1 = 0.50\Omega$, $R_2 = 0.20\Omega$, $\mathcal{E} = 12.0V$, $\mathcal{E}' = 6.0V$, $R_i = 0.025\Omega$, and $R_i' = 0.02\Omega$, find the currents in the resistances R_1 and R_2.

$$\begin{cases} R_i' I_i' + R_1 I_1 = \mathcal{E}' \\ (R_i + R_2) I_2 + R_1 I_1 = \mathcal{E} \\ I_i' + I_2 = I_1 \end{cases} \implies \begin{cases} 0.02 I_i' + 0.5 I_1 = 6 \\ 0.225 I_2 + 0.5 I_1 = 12 \\ I_i' + I_2 = I_1 \end{cases} \implies \begin{cases} 0.52 I_i' + 0.5 I_2 = 6 \\ 0.5 I_i' + 0.725 I_2 = 12 \\ I_i' + I_2 = I_1 \end{cases}$$

$$\therefore I_1 = \frac{1590}{127} \approx 12.5A \text{ and } I_2 = \frac{3240}{127} \approx 25.5A$$