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ENGS141 Engineering Statics - Homework 3

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Problem 3.81 (10 points).

3.81 A 500-N force is applied to a bent plate as shown. Determine (*a*) an equivalent force-couple system at *B*, (*b*) an equivalent system formed by a vertical force at *A* and a force at *B*.

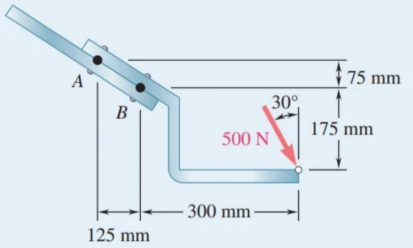
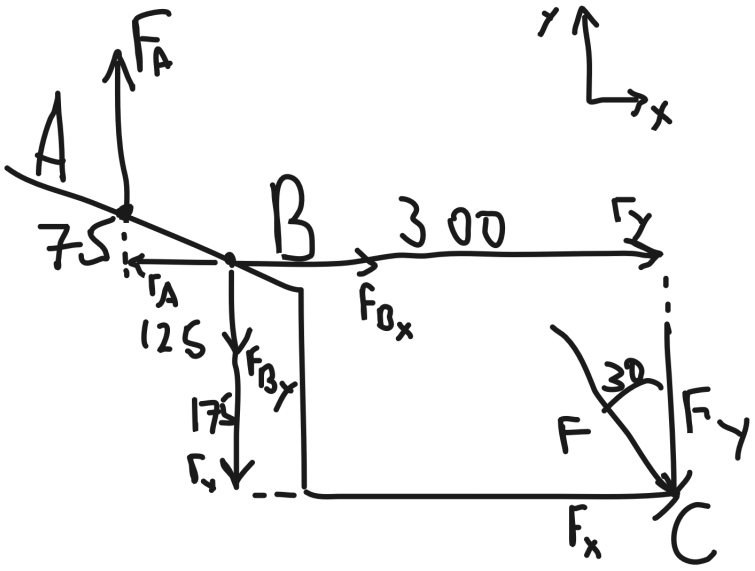


Fig. P3.81



All distance measurements are in *mm* and angles in °.

Break down the $500N$ force into two perpendicular forces in horizontal and vertical directions, and separately calculate and add the moments.

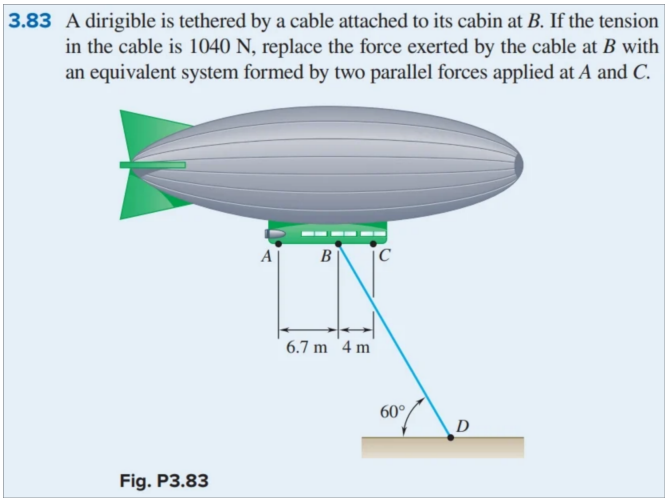
$$a)\mathbf{M}_B = \mathbf{r}_C \times \mathbf{F}_C = r_x F_x + r_y F_y = 0.175 \cdot 500 \sin 30^\circ - 0.300 \cdot 500 \cos 30^\circ \approx -86.15 \hat{\boldsymbol{k}} Nm$$

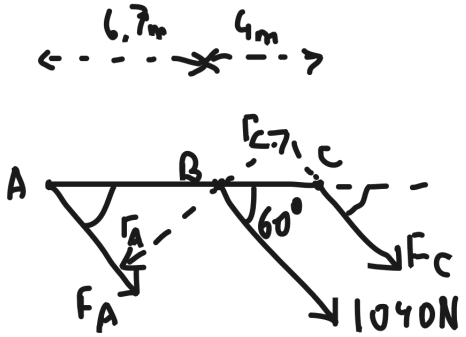
The equivalent force is the same $500N$ force as given in the problem.

$$b)\mathbf{M}_B = \mathbf{r}_A \times \mathbf{F}_A = 0.125 F_A \implies F_A \approx 689.23N$$

$$F_A - F_{By} = -F_y, F_B x = F_x \implies \mathbf{F}_B \approx 250\hat{\boldsymbol{\imath}} - 1122.24\hat{\boldsymbol{\jmath}} N$$

Problem 3.83 (10 points).





$$F_A + F_C = 1040, F_A r_A - F_C r_C = 0 \implies \begin{cases} F_A + F_C = 1040 \\ 6.7 \cos 30^\circ F_A = 4 \cos 30^\circ F_C \end{cases} \implies F_A \approx 388.79N, F_C \approx 651.21N$$

Both forces are directed 60 degrees below the horizon.

Problem 3.96 (10 points).

3.96 To keep a door closed, a wooden stick is wedged between the floor and the doorknob. The stick exerts at *B* a 175-N force directed along line *AB*. Replace that force with an equivalent force-couple system at *C*.

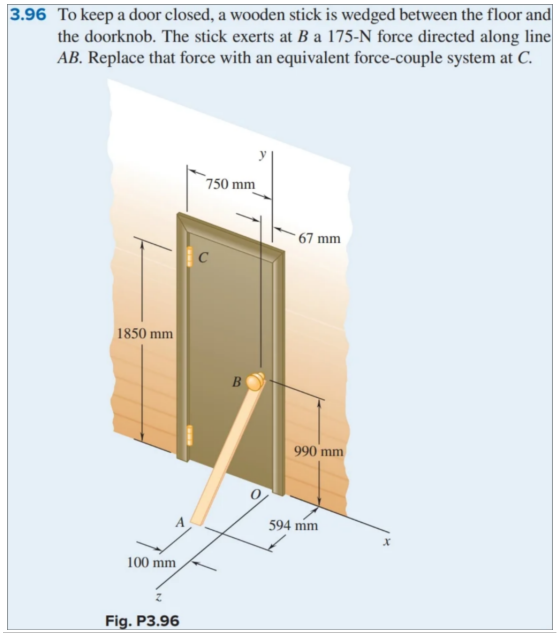
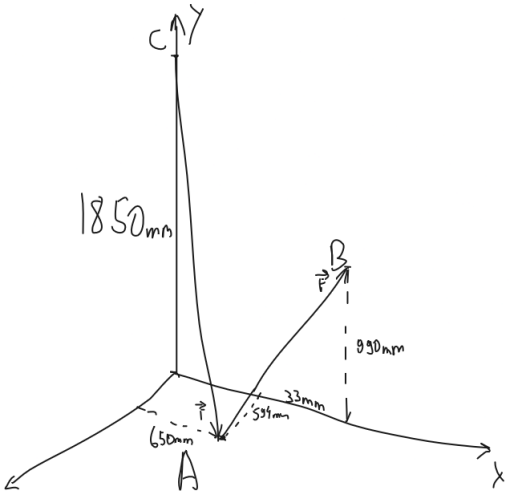


Fig. P3.96



$$AB = \sqrt{0.033^2 + 0.990^2 + 0.594^2} = 1.155m, F_{AB} = 175N\left(\frac{0.033}{1.155}\hat{\boldsymbol{\imath}} + \frac{0.990}{1.155}\hat{\boldsymbol{\jmath}} - \frac{0.594}{1.155}\hat{\boldsymbol{k}}\right) = 5\hat{\boldsymbol{\imath}} + 150\hat{\boldsymbol{\jmath}} - 90\hat{\boldsymbol{k}}N$$

$$\mathbf{r} = 0.650\hat{\boldsymbol{\imath}} - 1.850\hat{\boldsymbol{\jmath}} + 0.594\hat{\boldsymbol{k}}m \implies \mathbf{M}_C = \mathbf{r} \times \mathbf{F} = \begin{bmatrix} 0.650 \\ -1.850 \\ 0.594 \end{bmatrix} \times \begin{bmatrix} 5 \\ 150 \\ -90 \end{bmatrix} = 77.4\hat{\boldsymbol{\imath}} + 61.47\hat{\boldsymbol{\jmath}} + 106.75\hat{\boldsymbol{k}}Nm$$

Problem 3.101 (10 points).

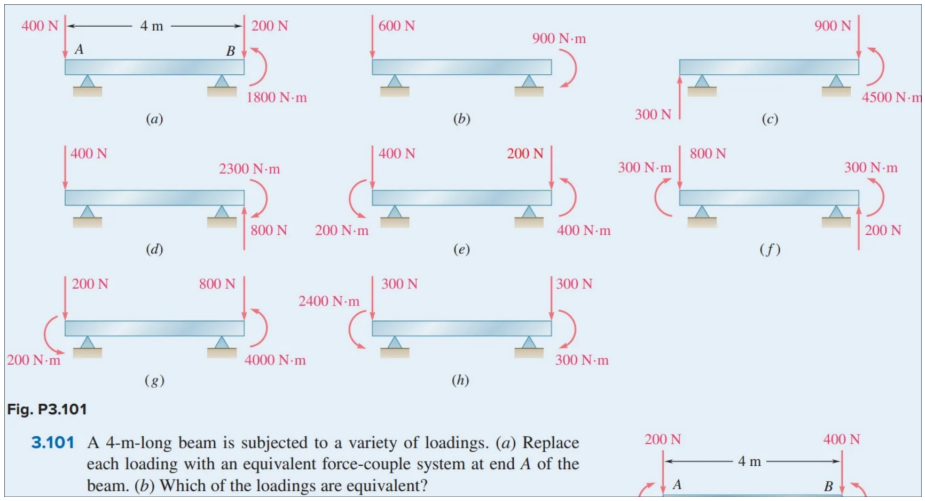
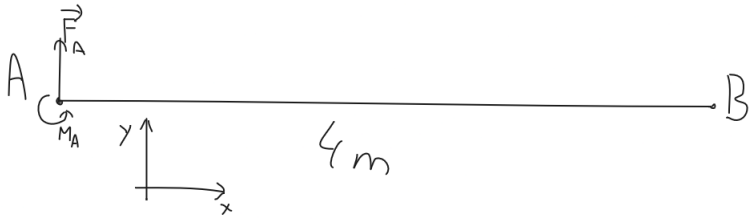


Fig. P3.101

3.101 A 4-m-long beam is subjected to a variety of loadings. (*a*) Replace each loading with an equivalent force-couple system at end *A* of the beam. (*b*) Which of the loadings are equivalent?



a) $M_A = 1800 - 4 \cdot 200 = 1000Nm, F_A = -200 - 400 = -600N$

b) $M_A = -900Nm, F_A = -600N$

c) $M_A = 4500 - 4 \cdot 900 = 900Nm, F_A = -600N$

d) $M_A = -2300 + 4 \cdot 800 = 900Nm, F_A = 400N$

e) $M_A = 200 + 400 - 4 \cdot 200 = -200Nm, F_A = -600N$

f) $M_A = -300 + 300 + 4 \cdot 200 = 800Nm, F_A = -600N$

g) $M_A = 200 + 4000 - 4 \cdot 800 = 1000Nm, F_A = -1000N$

h) $M_A = 2400 - 300 - 4 \cdot 300 = 900Nm, F_A = -600N$

Therefore c) and h) are equivalent.

Problem 3.116 (10 points).

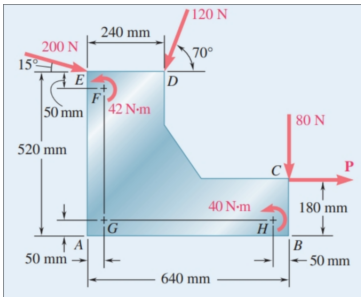


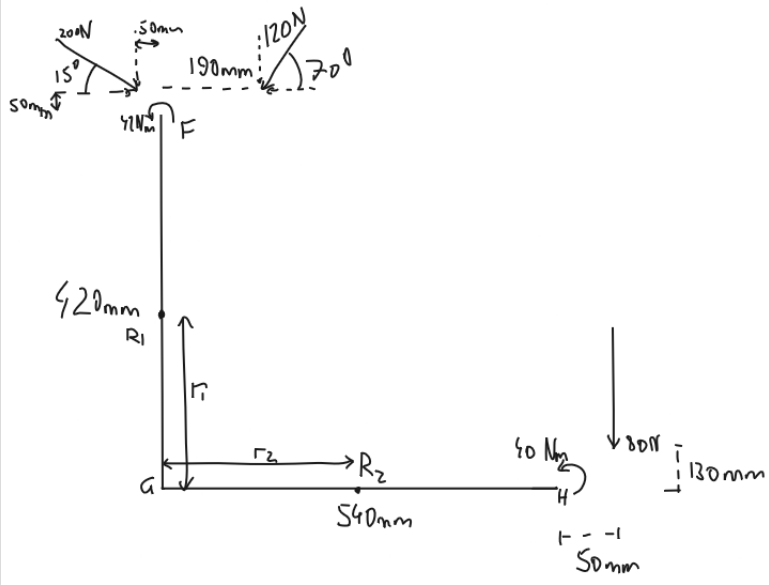
Fig. P3.116

3.116 A machine component is subjected to the forces and couples shown. The component is to be held in place by a single rivet that can resist a force but not a couple. For $P = 0$, determine the location of the rivet hole if it is to be located (*a*) on line *FG*, (*b*) on line *GH*.

3.117 Solve Prob. 3.116, assuming that $P = 60$ N.

3.118 As follower *AB* rolls along the surface of member *C*, it exerts a constant force **F** perpendicular to the surface. (*a*) Replace **F** with an equivalent force-couple system at the point *D* obtained by drawing the perpendicular from the point of contact to the *x* axis. (*b*) For $a = 1$ m and $b = 2$ m, determine the value of *x* for which the moment of the equivalent force-couple system at *D* is maximum.

3.119 A machine component is subjected to the forces shown, each of



Replace all the forces and moments with a force-couple system at G.

$$\sum F_x = 200\cos 15|circ - 120\cos 70^\circ \approx 152.14N$$

$$\sum F_y = -80 - 200\sin 15^\circ - 120\sin 70^\circ \approx -244.53N$$

$$M_G = 40+42+0.47\cdot(-200\cos 15^\circ+120\cos 70^\circ)+0.050\cdot 200\sin 15^\circ-0.19\cdot 120\sin 70^\circ-0.59\cdot 80 \approx -55.54Nm$$

$$r_1 = \frac{55.54}{152.14} \approx 0.364m, r_1 = \frac{55.54}{244.53} \approx 0.227m$$

Problem 3.131(10 points).

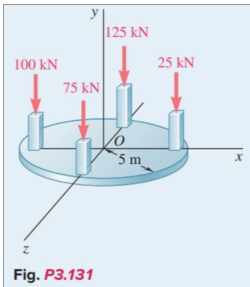
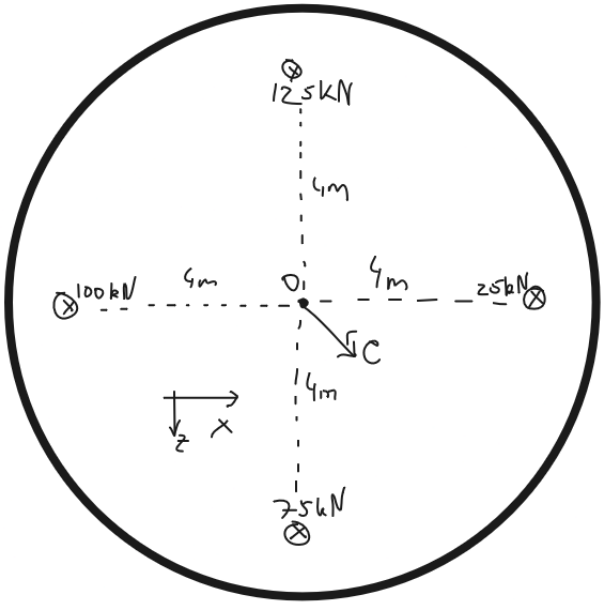


Fig. P3.131

Fig. P3.129 and P3.130

3.130 Four signs are mounted on a frame spanning a highway, and the magnitudes of the horizontal wind forces acting on the signs are as shown. Determine *a* and *b* so that the point of application of the resultant of the four forces is at *G*.

3.131 A concrete foundation mat of 5-m radius supports four equally spaced columns, each of which is located 4 m from the center of the mat. Determine the magnitude and the point of application of the resultant of the four loads.



$$\mathbf{M}_O = 4(75000 - 125000)\hat{\boldsymbol{\imath}} + 4(100000 - 25000)\hat{\boldsymbol{k}} = -200000\hat{\boldsymbol{\imath}} + 300000\hat{\boldsymbol{k}} Nm$$

$$\mathbf{R} = (-125000 - 100000 - 25000 - 75000)\hat{\boldsymbol{\jmath}} = -325000\hat{\boldsymbol{\jmath}} N$$

$$-200000\hat{\boldsymbol{\imath}} + 300000\hat{\boldsymbol{k}} = \mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{bmatrix} C_x \\ 0 \\ C_z \end{bmatrix} \times \begin{bmatrix} 0 \\ -325000 \\ 0 \end{bmatrix} = 325000C_z\hat{\boldsymbol{\imath}} - 325000C_x\hat{\boldsymbol{k}}$$

$$\therefore C_x = \frac{300000}{-325000} \approx -0.923m, C_z = \frac{-200000}{325000} \approx -0.615m$$

Problem 3.100 (20 points).

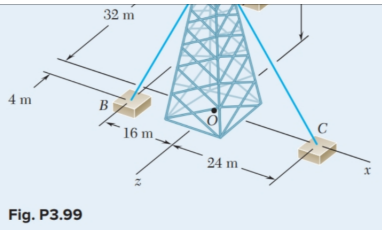


Fig. P3.99

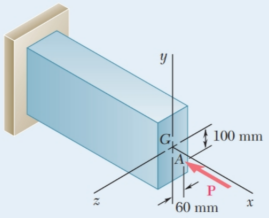


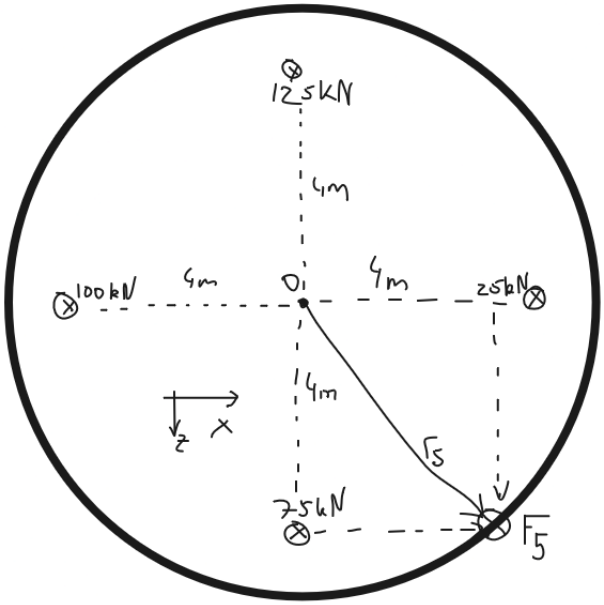
Fig. P3.100

3.100 An eccentric, compressive 1220-N force **P** is applied to the end of a cantilever beam. Replace **P** with an equivalent force-couple system at *G*.

$$\mathbf{M}_G = \mathbf{r} \times \mathbf{P} = \begin{bmatrix} 0 \\ -0.100 \\ -0.060 \end{bmatrix} \times \begin{bmatrix} -1220 \\ 0 \\ 0 \end{bmatrix} = 73.2\hat{\boldsymbol{\jmath}} - 122\hat{\boldsymbol{k}} Nm, \mathbf{F} = \mathbf{P} = -1220\hat{\boldsymbol{\imath}} N$$

Problem 3.132 (20 points).

3.132 Determine the magnitude and the point of application of the smallest additional load that must be applied to the foundation mat of Prob. 3.131 if the resultant of the five loads is to pass through the center of the mat.



Since we need the smallest force, it has to be on the edge of the circle, $5m$ away from the center, to provide maximum moment for its force. The moment for the other forces were taken from the solution of 3.131:

$$M_O = (-200000 + r_{5z}F_5)\hat{\boldsymbol{i}} + (300000 - r_{5x}F_5)\hat{\boldsymbol{k}} = 0 \text{ and } r_{5x}^2 + r_{5z}^2 = 5^2$$

$$\begin{cases} r_{5z}F_5 = 200000 \\ r_{5x}F_5 = 300000 \\ r_{5z} = \sqrt{25 - r_{5x}^2} \end{cases} \implies \frac{\sqrt{25 - r_{5x}^2}}{r_{5x}} = \frac{2}{3} \implies r_{5x} \approx 4.16m, r_{5z} \approx 2.77m, F_5 \approx 72100N$$

Python solutions

```
import numpy as np

def Problem3_81():
    # Values
    xBA = -0.125
    yBA = 0.075

    Force = 500
    Angle = 30
    xForceArm = 0.300
    yForceArm = -0.175

    BAlength = np.sqrt(xBA**2+yBA**2)
    BA = [xBA/BAlength, yBA/BAlength,0]

    vector_arm = [xForceArm,yForceArm,0]
    vector_force = [Force*np.sin(Angle* np.pi / 180),-Force*np.cos(Angle* np.pi / 180),0]
    moment = np.cross(vector_arm,vector_force)
    Force_A = moment[2]/xBA
    return ((moment, vector_force),(Force_A, [vector_force[0],vector_force[1]-Force_A,0]))
```

```
def Problem3_83():
    # Values

    Force = 1040
    Angle = 60

    AB = 6.7
    BC = 4

    ForceC = Force/(1+BC/AB)
    ForceA = Force-ForceC

    return ((ForceA, Angle), (ForceC, Angle))

def Problem3_96():
    # Values

    Force = 175
    Force_x = 0.100-0.067
    Force_y = 0.990
    Force_z = -0.594

    Arm_x = 0.750-0.100
    Arm_y = -1.850
    Arm_z = 0.594

    length = np.sqrt(Force_x**2 + Force_y**2+Force_z**2)
    vector_force = [Force*Force_x/length,Force*Force_y/length,Force*Force_z/length]
    vector_arm = [Arm_x, Arm_y, Arm_z]

    return np.cross(vector_arm,vector_force)


print("Problem 3.81: ", Problem3_81())
print("Problem 3.83: ", Problem3_83())
print("Problem 3.96: ", Problem3_96())
```