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ENGS141 Engineering Statics - Homework 5

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Problem 5.6

Locate the centroid of the plane area shown:

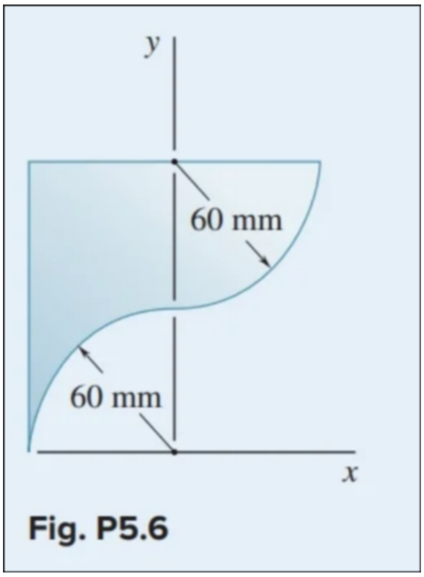

Fig. P5.6

Start with the rectangle $x \leq 0$, with side lengths $x = 60mm$, $y = 120mm$. Sutract the bottom quarter circle and add the top right quarter circle with radii $r = 60mm$. The total area of the final figure equals to the are of the rectangle in the beginning.

$$\begin{cases} x_r = -30mm, y_r = 60mm, S = 7200mm \\ x_{c1} = -\frac{80}{\pi}m, y_{c1} = \frac{80}{\pi}, S_{c1} = -900\pi mm^2 \\ x_{c1} = \frac{80}{\pi}m, y_{c1} = 120 - \frac{80}{\pi}, S_{c1} = 900\pi mm^2 \end{cases} \implies \begin{cases} Q_x = -72000mm^3 \\ Q_y = 627292mm^3 \end{cases} \implies \begin{cases} x_c = -10mm \\ y_c = 87mm \end{cases}$$

Problem 5.12

Locate the centroid of the plane area shown:

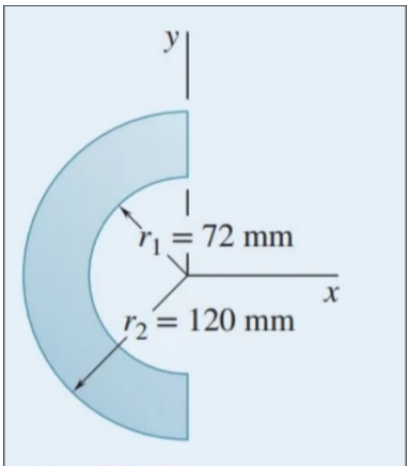

Fig. P5.12

Start with the larger semicircle then remove the the smaller semicircle. The toal area of the figure is $S = 9216\pi mm^2$.

$$\begin{cases} x_{c1} = -\frac{160}{\pi}mm, y_{c1} = 0mm, S_{c1} = 120^2\pi \\ x_{c2} = -\frac{96}{\pi}mm, y_{c2} = 0mm, S_{c2} = -72^2\pi \end{cases} \implies \begin{cases} Q_x = -1806336mm^3 \\ Q_y = 0 \end{cases} \implies \begin{cases} x_c = -62mm \\ y_c = 0mm \end{cases}$$

Problem 5.38

Determine by direct integration the centroid of the area shown:

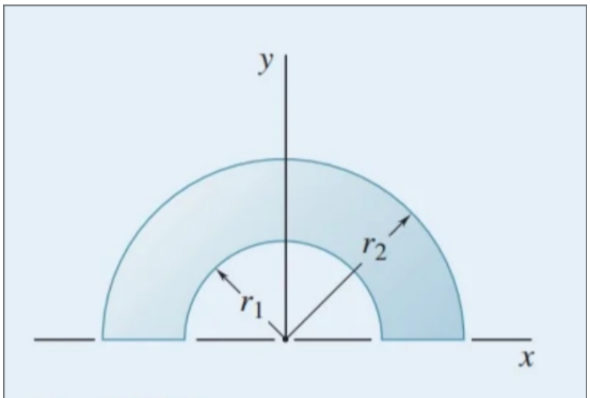

Fig. P5.38

$$Q_{x1} = \int_{-r_1}^{r_1} \int_0^{\sqrt{r_1^2 - x^2}} x \,\mathrm{d}y \,\mathrm{d}x = \int_{-r_1}^{r_1} x\sqrt{r_1^2 - x^2} \,\mathrm{d}x = 0$$

$$Q_{y1} = \int_{-r_1}^{r_1} \int_0^{\sqrt{r_1^2 - x^2}} y \,\mathrm{d}y \,\mathrm{d}x = \int_{-r_1}^{r_1} \frac{r_1^2 - x^2}{2} \,\mathrm{d}x = \frac{2r_1^3}{3}$$

$$Q_{x2} = \int_{-r_2}^{r_2} \int_0^{\sqrt{r_2^2 - x^2}} x \,\mathrm{d}y \,\mathrm{d}x = \int_{-r_2}^{r_2} x\sqrt{r_2^2 - x^2} \,\mathrm{d}x = 0$$

$$Q_{y2} = \int_{-r_2}^{r_2} \int_0^{\sqrt{r_2^2 - x^2}} y \,\mathrm{d}y \,\mathrm{d}x = \int_{-2_1}^{2_1} \frac{r_2^2 - x^2}{2} \,\mathrm{d}x = \frac{2r_2^3}{3}$$

$$\begin{cases} Q_y = \dfrac{2}{3}(r_2^3 - r_1^3) \\ S = \dfrac{\pi}{2}(r_2^2 - r_1^2) \end{cases} \implies \begin{cases} x_c = 0 \\ y_c = \dfrac{4}{3\pi} \cdot \dfrac{(r_2^3 - r_1^3)}{(r_2^2 - r_1^2)} \end{cases}$$

Problem 5.54

5.54 Determine the volume and the surface area of the solid obtained by rotating the area of Prob. 5.6 about (*a*) the line $x = -60$ mm, (*b*) the line $y = 120$ mm.

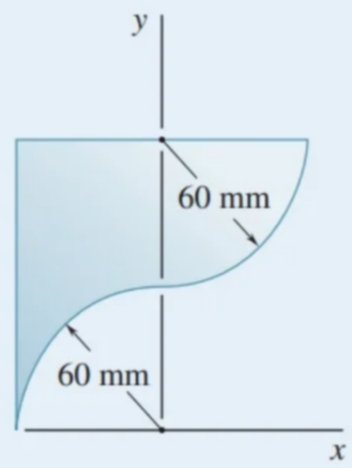

Fig. P5.6

$$V_1 = 2\pi(60 \cdot 120)(60 - 10) = 2.26 \cdot 10^6 mm^3$$

$$V_2 = 2\pi(60 \cdot 120)(120 - 87) = 1.49 \cdot 10^6 mm^3$$

The centroid of the curve is where the curve would balance, which is at $x = 0mm, y = 60mm$. Use the integral to find the surface area and add the area of the side circle with radius $r = 120mm$.

$$A_1 = 2\pi(2 \cdot \frac{2\pi}{4} \cdot 60)(60 - 0) + \pi 120^2 = 1.16 \cdot 10^5 mm^2$$

$$A_2 = 2\pi(2 \cdot \frac{2\pi}{4} \cdot 60)(120 - 60) + \pi 120^2 = 1.16 \cdot 10^5 mm^2$$

Problem 5.30

5.30 The homogeneous wire *ABCD* is bent as shown and is attached to a hinge at *C*. Determine the length *L* for which portion *BCD* of the wire is horizontal.

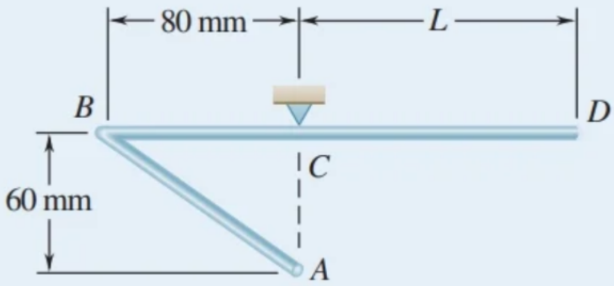

Fig. P5.30 and P5.31

Take the x axis on BD and y axis on AC. $AB = \sqrt{80^2 + 60^2} = 100mm$. Centroids of AB and BD are at the midpoints. The centroid of the whole construction should be along AC, at $x = 0$.

$$\begin{cases} xL_{AB} = 100 \cdot (-40) = -4000mm^2 \\ xL_{BC} = 80 \cdot (-40) = -3200mm^2 \\ xL_{CD} = L \cdot \left(\frac{L}{2}\right) = \frac{L^2}{2} \end{cases} \implies \frac{L^2}{2} = 7200 \implies L = 120mm$$

Problem 5.53

5.53 Determine the volume and the surface area of the solid obtained by rotating the area of Prob. 5.7 about (*a*) the *x* axis, (*b*) the *y* axis.

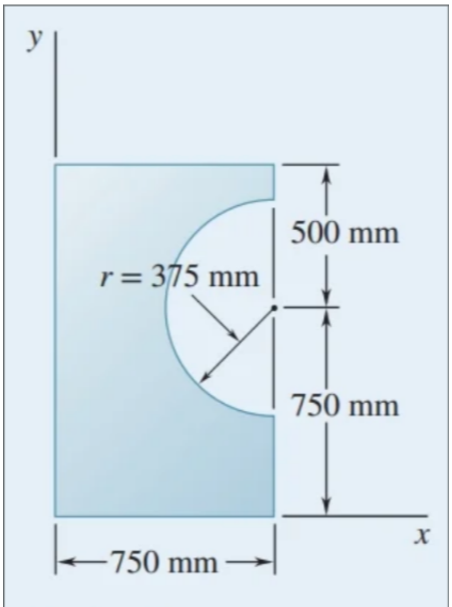

Fig. P5.7

Step 1, calculate the centroid of area. Take the rectangle, and remove the semicircle.

$$S = 0.750 * 1.250 - (pi * 0.375^2)/(2) \approx 0.716607m^2$$

$$\begin{cases} Q_x = 0.375 \cdot (0.750 \cdot 1.250) - \left(0.750 - \frac{4 \cdot 0.375}{3\pi}\right)\left(\frac{1}{2}\pi 0.375^2\right) \approx 0.22105m^3 \\ Q_y = 0.625 \cdot (0.750 \cdot 1.250) - (0.750)\left(\frac{1}{2}\pi 0.375^2\right) \approx 0.42027m^3 \end{cases} \implies \begin{cases} x_c \approx 0.308m \\ y_c \approx 0.586m \end{cases}$$

$$V_x = 2\pi S \cdot 0.308 \approx 1.39m^3$$

$$V_y = 2\pi S \cdot 0.586 \approx 2.64m^3$$

To get the surface area, first use the centeroid of the semicircle and the theorem to get the area generated by revolution of the semicircle, then add the other sides.

$$S_1 = 2\pi(\pi \cdot 0.375)(0.750 - \frac{2 \cdot 0.375}{\pi}) + 2\pi 0.750^2 + 2\pi \cdot 0.750 \cdot 0.500 \approx 9.67m^2$$

$$S_2 = 2\pi(\pi \cdot 0.375)(0.750) + 2\pi 1.250^2 + 2\pi \cdot 1.250 \cdot 0.750 - \pi 1.125^2 + \pi 0.375^2 \approx 17.73m^2$$

Problem 5.65

***5.65** The shade for a wall-mounted light is formed from a thin sheet of translucent plastic. Determine the surface area of the outside of the shade, knowing it has the parabolic cross section shown.

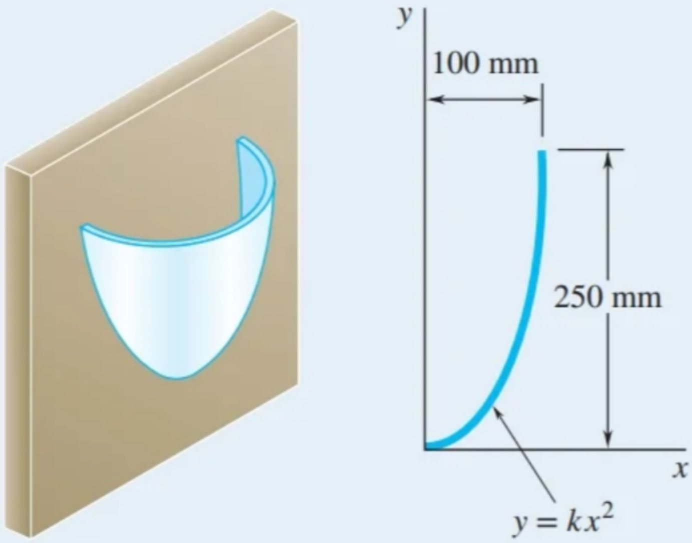

Fig. P5.65

$$0.250 = k0.100^2 \implies k = 25, y = kx^2, x = \sqrt{\frac{y}{k}} L = \int_0^{0.1} \sqrt{1 + (50x)^2}\, \mathrm{d}x \approx 0.2781 m$$

$$x_c = \frac{1}{L} \int_0^{0.1} x\sqrt{1 + (50x)^2}\, \mathrm{d}x \approx 0.0631 m, A = \pi \cdot 0.0631 \cdot 0.2781 \approx 0.0551 m^2$$