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ENGS141 Engineering Statics - Homework 6

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Problem 5.69

Determine the reactions at the beam supports for the given loading.

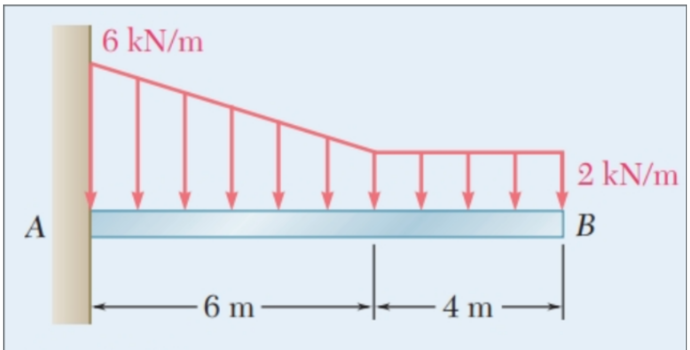

Fig. P5.69

$$F_1 = \frac{4 \cdot 6}{2} = 12kN, F_2 = 2 \cdot 6 = 12kN, F_3 = 2 \cdot 4 = 8kN$$

$$c_{x1} = 2m, c_{x2} = 3m, c_{x3} = 8m$$

$$\therefore F = 32kN, M = 12 \cdot 2 + 12 \cdot 3 + 8 \cdot 8 = 124kNm$$

The reaction force is directed upwards, and the moment is directed counter-clockwise.

Problem 5.73

Determine the reactions at the beam supports for the given loading.

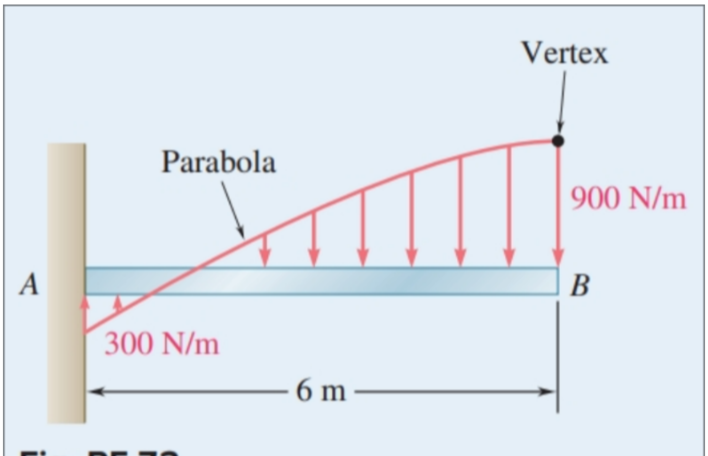

Fig. P5.73

$$f(0) = -300N/m, f(6) = 900N/m \implies f(x) = -\frac{100}{3}(x-6)^2 + 900$$

$$\therefore F = \int_0^6 f(x)\,\mathrm{d}x = 3000N, M = \int_0^6 xf(x)\,\mathrm{d}x = 12600Nm$$

The reaction force is directed upwards, and the moment is directed counter-clockwise.

Problem 5.99

5.99 Locate the centroid of the frustum of a right circular cone when $r_1 = 40$ mm, $r_2 = 50$ mm, and $h = 60$ mm.

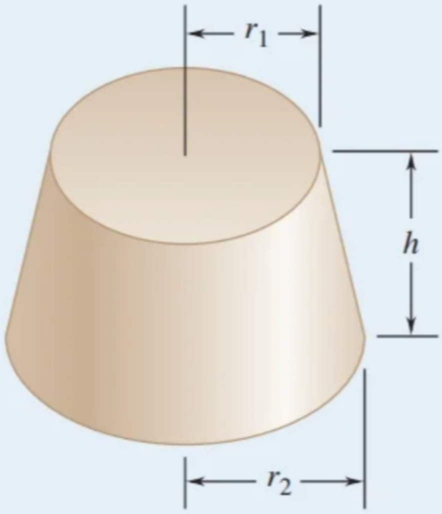

Fig. P5.99

It is trivial that the centeroid lies in the central axis of the cone. Complete the cone, and do subtraction of cones:

$$V_0 = \frac{1}{3}\pi r_2^2 h_0 = \frac{1}{3}\pi 50^2 300, V_1 = \frac{1}{3}\pi 40^2 240$$

$$Q_z = \frac{300}{4}\frac{1}{3}\pi 50^2 300 - \left(60 + \frac{240}{4}\right)\frac{1}{3}\pi 40^2 240$$

$$\therefore z_c = \frac{Q_z}{V_0 - V_1} = \frac{1695}{61} \approx 27.8mm$$

Wolframalpha was used to do the complicated calculations.

Problem 5.106

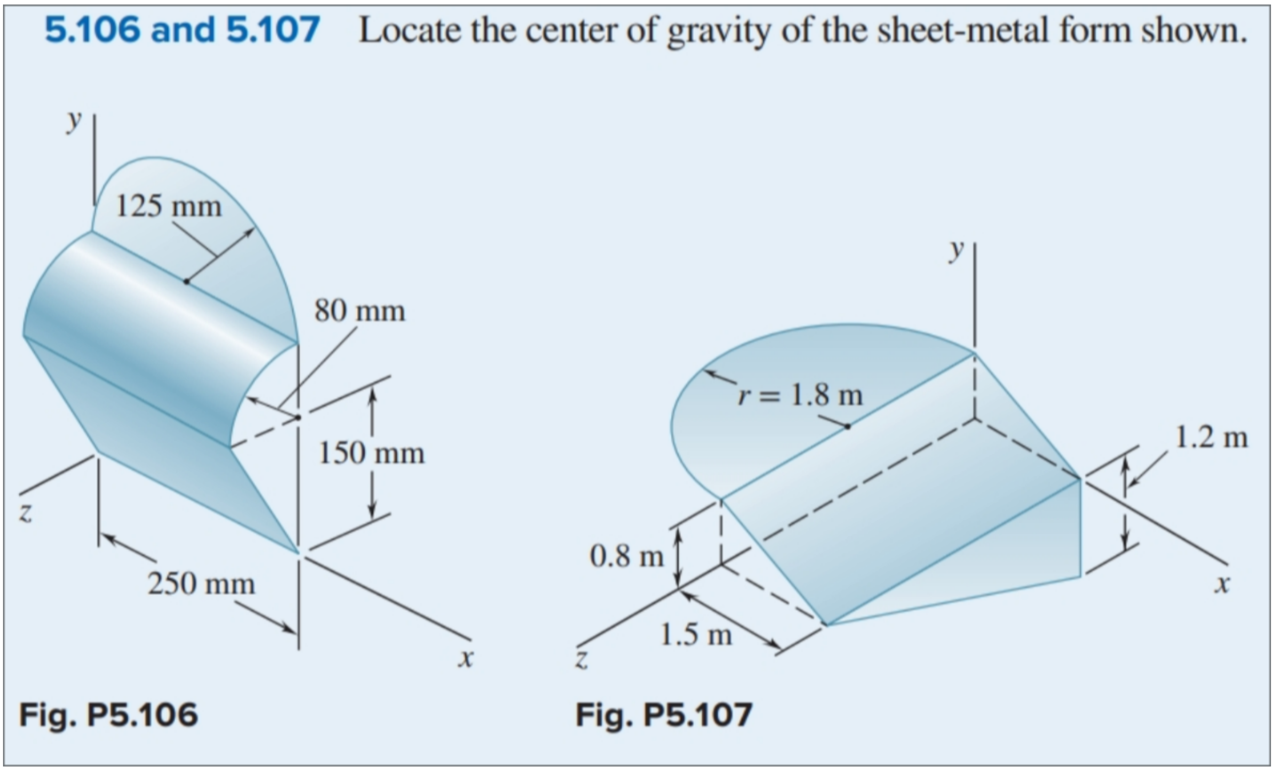

Fig. P5.106

Fig. P5.107

Area of rectangle is $S_1 = 250 \cdot 170 = 42500$, centroid is at $C_1(125, 75, 40)$.
Area of arc is $S_2 = 250 \cdot \frac{80\pi}{2} = 10000\pi$, centroid is at $C_2(125, 150 + \frac{160}{\pi}, \frac{160}{\pi})$.
Area of semicircle is $S_3 = \frac{\pi}{2}125^2 = 7812.5\pi$, centroid is at $C_3(125, 230 + \frac{500}{3\pi}, 0)$.

$$C = \frac{S_1C_1 + S_2C_2 + S_3C_3}{S_1 + S_2 + S_3} \approx (125.0, 167.0, 33.5)mm$$

Problem 5.126

Locate the centroid of the volume obtained by rotating the shaded area about the x axis.

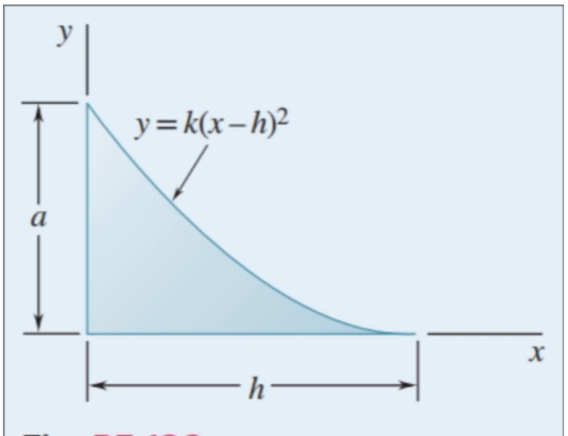

Fig. P5.126

It is trivial that the centroid is along the x axis.

$$a = kh^2,\ V = \int_0^h \pi(k(x-h)^2)^2\,\mathrm{d}x = \pi k^2 \int_0^h (x-h)^4\,\mathrm{d}x = \frac{\pi k^2 h^5}{5} = \frac{\pi a^2 h}{5}$$

$$Q_x = \int_0^h x\pi(k(x-h)^2)^2\,\mathrm{d}x = \pi k^2 \int_0^h x(x-h)^4\,\mathrm{d}x = \pi k^2 \int_{-h}^0 (x+h)x^4\,\mathrm{d}x = \pi k^2 \left(\frac{h^6}{5} - \frac{h^6}{6}\right) = \frac{\pi a^2 h^2}{30}$$

$$\therefore x_c = \frac{h}{6}$$