Digital version

ENGS241 Computer Aided Design - Homework 3

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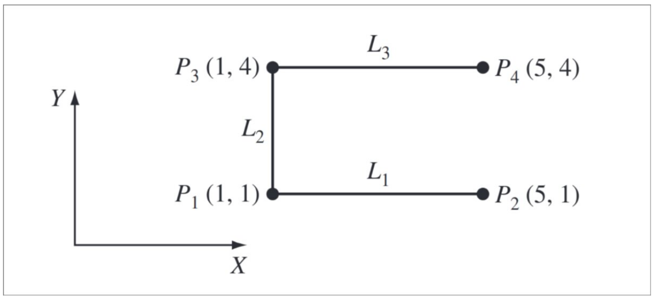
Problem 6.1

Find the equations of the three lines L_1, L_2, and L_3 shown. Are L_1 and L_2 perpendicular? Are L_1 and L_3 parallel? Prove your answers.



$$L_1 : \mathbf{r} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix} s, s \in [0, 1]$$

$$L_2 : \mathbf{r} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} t, t \in [0, 1]$$

$$L_3 : \mathbf{r} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix} u, u \in [0, 1]$$

$$\begin{bmatrix} 4 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 3 \end{bmatrix} = 4 \cdot 0 + 0 \cdot 3 = 0 \implies L_1 \perp L_2$$

$$\begin{bmatrix} 4 \\ 0 \end{bmatrix} = 1 \cdot \begin{bmatrix} 4 \\ 0 \end{bmatrix} \implies L_1 \parallel L_3$$

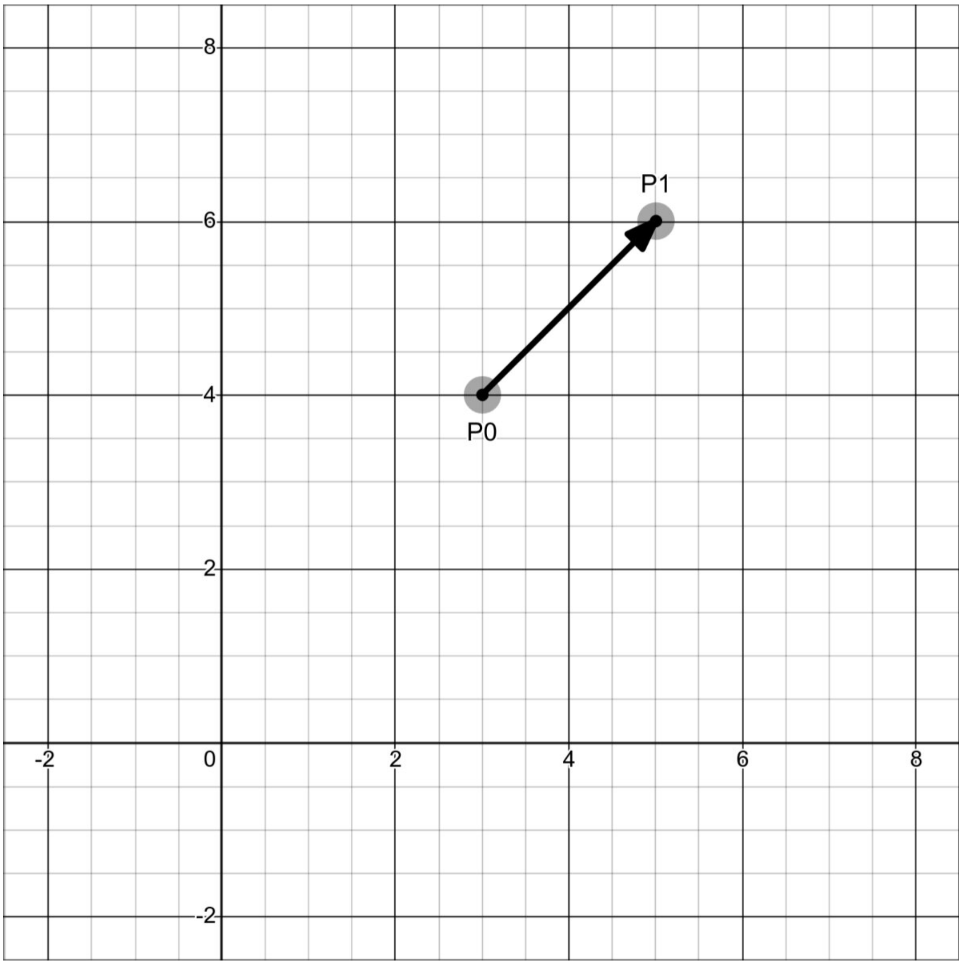
Problem 6.2

A line equation is given by:

$$\mathbf{P} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} + u \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

Find the end points of the line. Sketch the line with the end points and the u direction. Find the intersection points between the line and a circle with a center at $(1, 2, 0)$ and radius of 2.

It was assumed that $u \in [0, 1]$. Therefore, the endpoints are $P_0 = (3, 4, 0)$ and $P_1 = (5, 6, 0)$.



The parametric equation for the line and the circle are:

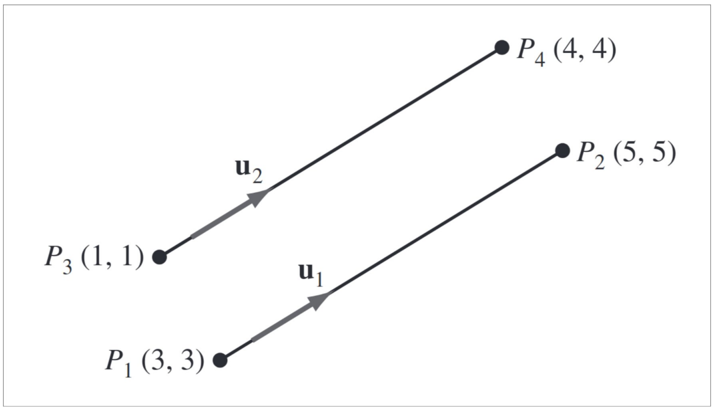
$$\begin{cases} x = 3 + 2u \\ y = 4 + 2u \end{cases}, u \in [0, 1] \text{ and } \begin{cases} x = 1 + 2\cos(2\pi t) \\ y = 2 + 2\sin(2\pi t) \end{cases}, t \in [0, 1]$$

$$\begin{cases} 1 + u = \cos(2\pi t) \\ 1 + u = \sin(2\pi t) \end{cases} \implies 2u^2 + 4u + 1 = 0$$

$$u = \frac{-4 \pm \sqrt{16 - 8}}{4} = \frac{-4 \pm 2\sqrt{2}}{4} \notin [0, 1] \implies \text{ no intersection}$$

Problem 6.3

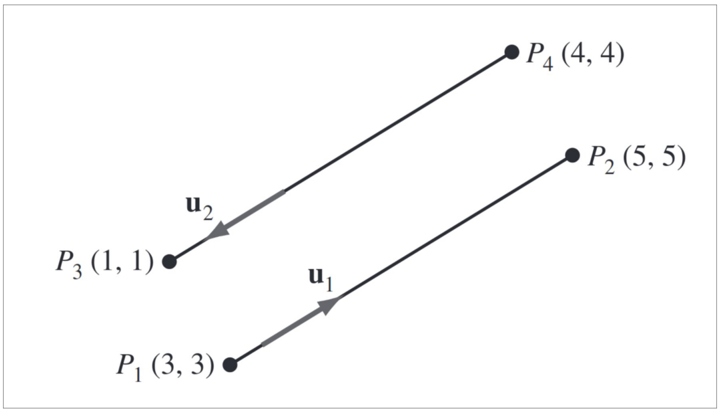
Find the angle θ between the two lines for both cases shown below.



$$L_1 : \mathbf{r} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} s, s \in [0, 1]$$

$$L_2 : \mathbf{r} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \end{bmatrix} t, t \in [0, 1]$$

$$\cos\theta = \frac{\begin{bmatrix} 2 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 3 \end{bmatrix}}{\left\| \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\| \left\| \begin{bmatrix} 3 \\ 3 \end{bmatrix} \right\|} = 1 \implies \theta = 0$$



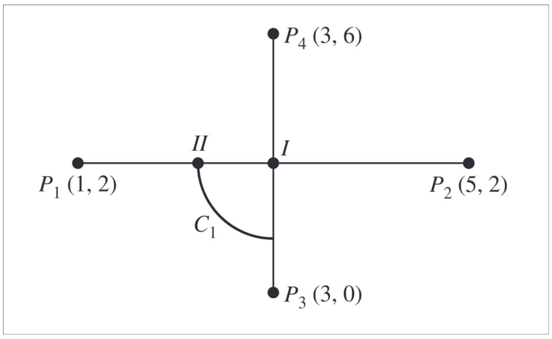
$$L_1 : \mathbf{r} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} s, s \in [0, 1]$$

$$L_2 : \mathbf{r} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} + \begin{bmatrix} -3 \\ -3 \end{bmatrix} t, t \in [0, 1]$$

$$\cos\theta = \frac{\begin{bmatrix} 2 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ -3 \end{bmatrix}}{\left\| \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\| \left\| \begin{bmatrix} -3 \\ -3 \end{bmatrix} \right\|} = -1 \implies \theta = \pi$$

Problem 6.4

Two lines with their end points and an arc are shown bellow:



a) Find the equations of the lines.

$$\overrightarrow{P_1P_2} = \begin{bmatrix}1\\2\end{bmatrix} + t_1\begin{bmatrix}4\\0\end{bmatrix}, t_1 \in [0,1]$$

$$\overrightarrow{P_3P_4} = \begin{bmatrix}3\\0\end{bmatrix} + t_2\begin{bmatrix}0\\6\end{bmatrix}, t_2 \in [0,1]$$

b) Using the equations, find the intersection point I of the two lines.

$$\begin{bmatrix}1\\2\end{bmatrix} + t_1\begin{bmatrix}4\\0\end{bmatrix} = \begin{bmatrix}3\\0\end{bmatrix} + t_2\begin{bmatrix}0\\6\end{bmatrix} \implies t_1 = \frac{1}{2} \text{ and } t_2 = \frac{1}{3}$$

$$\therefore I = (3, 2)$$

c) Find the parametric equation of the arc C_1 whose center is I and whose radius is 1 inch.

$$\begin{cases} x = 3 + \cos(2\pi t_3) \\ x = 2 + \sin(2\pi t_3) \end{cases}, t_3 \in [a, b]$$

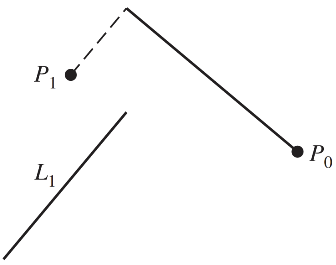
d) Find the intersection point II between C_1 and L_1.

$$L_1 : \begin{cases} x = 1 + 4t_1 \\ y = 2 \end{cases} \implies \begin{cases} 1 + 4t_1 = 3 + \cos(2\pi t_3) \\ 2 = 2 + \sin(2\pi t_3) \end{cases} \implies \begin{cases} t_3 = 0, \implies t_1 = \frac{3}{4} > \frac{1}{2} \\ t_3 = \pi \implies t_1 = \frac{1}{4} \end{cases}$$

$$\therefore II = (2, 2)$$

Problem 6.8 d

Find the equation and end points of a line perpendicular to L_1, passes through P_0, and bounded by P_1.



Take the direction vector of L_1 to be $\vec{d} = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$. The direction vector of our line will be $\vec{d'} = \begin{bmatrix} -d_y \\ d_x \end{bmatrix}$. Denote a point P_2, that lines on our line, such that $P_1P_2 \perp P_0P_2 \implies P_1P_2 \parallel L_1$. If we combine all these together, we get the following system of equations:

$$u_1 \begin{bmatrix} d_x \\ d_y \end{bmatrix} + \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = u_2 \begin{bmatrix} -d_y \\ d_x \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, u_1, u_2 \in \mathbb{R}$$

$$\begin{cases} u_1 d_x + x_1 = -u_2 d_y + x_0 \\ u_1 d_y + y_1 = u_2 d_x + y_0 \end{cases} \implies \begin{cases} u_1 = \dfrac{-u_2 d_y + x_0 - x_1}{d_x} \\ \dfrac{-u_2 d_y + x_0 - x_1}{d_x} d_y + y_1 = u_2 d_x + y_0 \end{cases}$$

$$\therefore u_2 = \frac{x_0 - x_1 d_y + y_1 d_x - y_0 d_x}{d_x^2 + d_y}$$

$$L_2 : \vec{r} = u_2 \begin{bmatrix} -d_y \\ d_x \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, u_2 \in \left[0, \frac{x_0 - x_1 d_y + y_1 d_x - y_0 d_x}{d_x^2 + d_y}\right]$$

The endpoints of our line segment are the point P_0 and the point we get, when we plug in the value $\dfrac{x_0 - x_1 d_y + y_1 d_x - y_0 d_x}{d_x^2 + d_y}$ in the above equation:

$$P_2 = \left(-d_y \frac{x_0 - x_1 d_y + y_1 d_x - y_0 d_x}{d_x^2 + d_y} + x_0, d_x \frac{x_0 - x_1 d_y + y_1 d_x - y_0 d_x}{d_x^2 + d_y} + y_0\right)$$

Phone Holder

The phone holder consists of two parts, the base and the pin. The pin has four possible slots to go in, that vary the angle of the phone. The charging port is cut out, alongside with the speaker grills. Designed for iPhone SE 2020, but compatible with many other smartphones.