Digital version

ENGS241 Computer Aided Design - Homework 4

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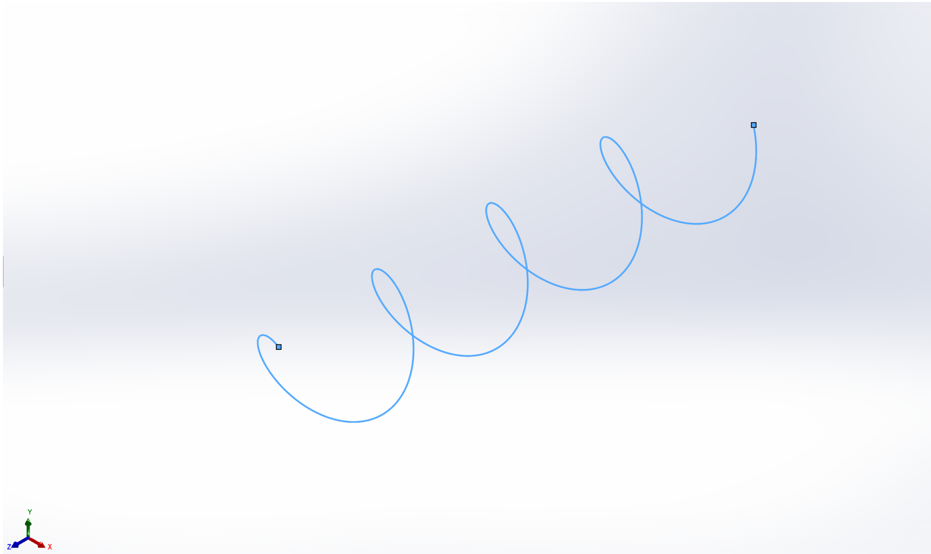
Helix and Bezier curve

$$\bar{P} = \begin{bmatrix} 2\cos\left(2\pi u - \frac{\pi}{4}\right) \\ -2\sin\left(2\pi u - \frac{\pi}{4}\right) \\ 5u \end{bmatrix}, 0 \leq u \leq \frac{31}{8}$$

a) Create it in SolidWorks;



b) Find the parametric equation of the line connecting the endpoints of the helix;

$$P_0 = \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \\ 0 \end{bmatrix}, P_1 = \begin{bmatrix} 0 \\ 2 \\ \frac{155}{8} \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \\ 0 \end{bmatrix} + \begin{bmatrix} -\sqrt{2} \\ 2-\sqrt{2} \\ \frac{155}{8} \end{bmatrix} t, t \in [0, 1]$$

c) Calculate the angle between the line connecting the endpoints of the helix and z axis;

$$\cos\varphi = \frac{\begin{bmatrix} -\sqrt{2} \\ 2-\sqrt{2} \\ \frac{155}{8} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}{\left\| \begin{bmatrix} -\sqrt{2} \\ 2-\sqrt{2} \\ \frac{155}{8} \end{bmatrix} \right\| \left\| \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\|} = \frac{\frac{155}{8}}{\sqrt{2 + (2-\sqrt{2})^2 + \frac{155^2}{8^2}}} \implies \varphi \approx 4.52^\circ$$

d) Find the tangent to the helix at its parametric midpoint;

$$\dot{P} = \begin{bmatrix} -4\pi \sin\left(2\pi u - \frac{\pi}{4}\right) \\ -4\pi \cos\left(2\pi u - \frac{\pi}{4}\right) \\ 5 \end{bmatrix}, P\left(\frac{31}{16}\right) = \begin{bmatrix} 2\cos\left(\frac{29}{8}\pi\right) \\ -2\sin\left(\frac{29}{8}\pi\right) \\ \frac{155}{16} \end{bmatrix}, \dot{P}\left(\frac{31}{16}\right) = \begin{bmatrix} -4\pi \sin\left(\frac{29}{8}\pi\right) \\ -4\pi \cos\left(\frac{29}{8}\pi\right) \\ 5 \end{bmatrix}$$

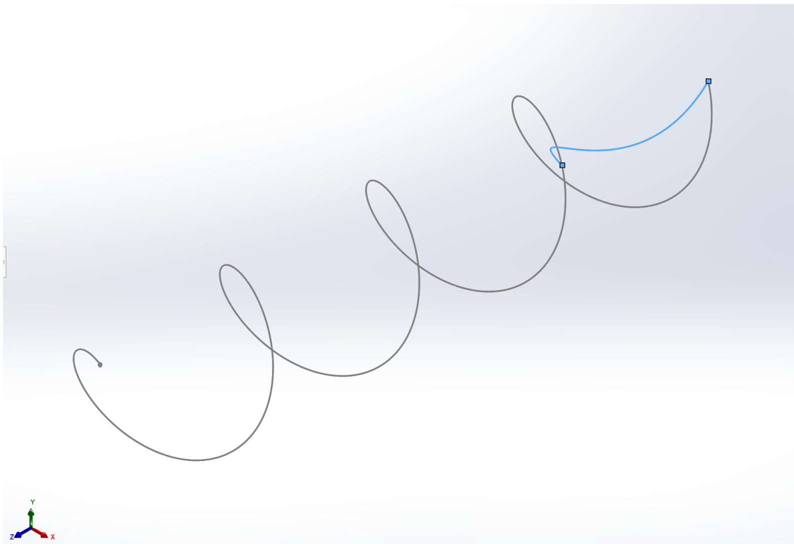
$$\vec{r} = \begin{bmatrix} 2\cos\left(\frac{29}{8}\pi\right) \\ -2\sin\left(\frac{29}{8}\pi\right) \\ \frac{155}{16} \end{bmatrix} + \begin{bmatrix} -4\pi \sin\left(\frac{29}{8}\pi\right) \\ -4\pi \cos\left(\frac{29}{8}\pi\right) \\ 5 \end{bmatrix} t, t \in \mathbb{R}$$

e) Find the parametric equation of Bezier curve with control points at $P(0)$, $P(1/3)$, $P(2/3)$, $P(1)$ and create the curve in SolidWorks.

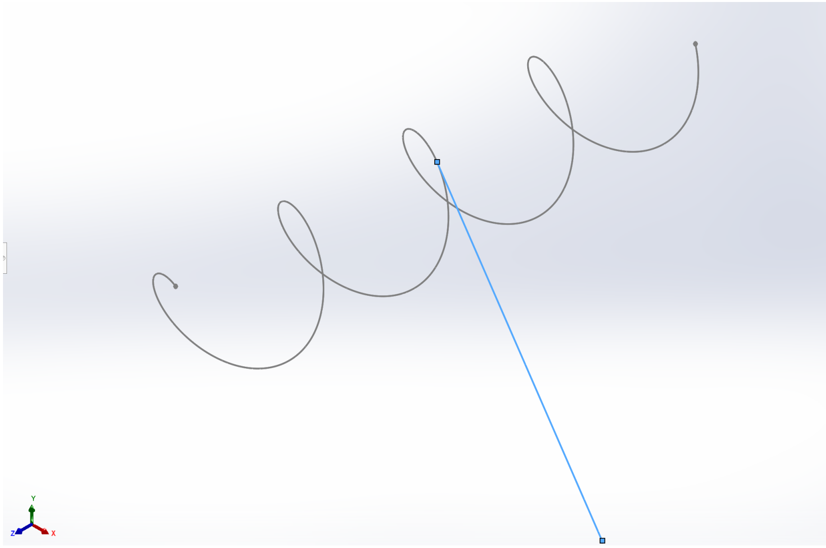
$$P(0) = \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \\ 0 \end{bmatrix}, P(1/3) = \begin{bmatrix} 2\cos\left(\frac{5}{12}\pi\right) \\ -2\sin\left(\frac{5}{12}\pi\right) \\ \frac{5}{3} \end{bmatrix}, P(2/3) = \begin{bmatrix} 2\cos\left(\frac{13}{12}\pi\right) \\ -2\sin\left(\frac{13}{12}\pi\right) \\ \frac{10}{3} \end{bmatrix}, P(1) = \begin{bmatrix} 2\cos\left(\frac{7}{4}\pi\right) \\ -2\sin\left(\frac{7}{4}\pi\right) \\ 5 \end{bmatrix}$$

$$B(t) = (1-t)^3 P_1 + 3(1-t)^2 t P_2 + 3(1-t)t^2 P_3 + t^3 P_4, t \in [0,1]$$

$$\begin{cases} x(t) = \sqrt{2}(1-t)^3 + 6(1-t)^2 t \cos\left(\frac{5}{12}\pi\right) + 6(1-t)t^2 \cos\left(\frac{13}{12}\pi\right) + 2t^3 \cos\left(\frac{7}{4}\pi\right) \\ x(t) = \sqrt{2}(1-t)^3 - 6(1-t)^2 t \sin\left(\frac{5}{12}\pi\right) - 6(1-t)t^2 \sin\left(\frac{13}{12}\pi\right) - 2t^3 \sin\left(\frac{7}{4}\pi\right) \\ z(t) = 5(1-t)^2 t + 10(1-t)t^2 + 5t^3 \end{cases}, t \in [0,1]$$



f) Create the vector found in (d) as a 3D line in your SolidWorks helix model.



Angular fitting