Digital version

ENGS252 Signals and Systems - Homework 3

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February 24, 2026

Problem 1

If the waveform is periodic, what is its period? The sampling rate for the signal is $T_s = 0.1$s.

a) $x[n] = 2 + 4\cos[0.7\pi n T_s]$

$$x[n] = 2 + 4\cos[0.07\pi n] \implies 0.07\pi n = 2\pi k \implies n = \frac{200}{7}k \overset{k=7}{=} 200$$

b) $x[n] = 2\cos[\sqrt{2}\pi n T_s]$

$$x[n] = 2\cos[0.1\sqrt{2}\pi n] \implies 0.1\sqrt{2}\pi n = 2\pi k \implies n = 10\sqrt{2}k$$

The waveform is not periodic, since there is no integer k for which n is an integer.

c) $x[n] = 5\cos\left[\sqrt{5}\pi n T_s + \frac{\pi}{3}\right]$

$$x[n] = 5\cos\left[0.1\sqrt{5}\pi n + \frac{\pi}{3}\right] \implies 0.1\sqrt{5}\pi n = 2\pi k \implies n = \frac{20}{\sqrt{5}}k$$

The waveform is not periodic, since there is no integer k for which n is an integer.

Problem 2

Find the minimum sampling rate required to represent the continuous signal unambiguously.

a) $x(t) = 3\cos\left(10\pi t + \frac{\pi}{3}\right)$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = 0.2s \implies T_s \leq 0.1s$$

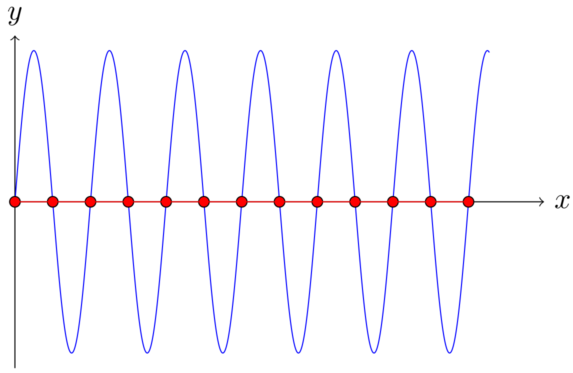
b) $x(t) = 3\cos\left(\sqrt{3}\pi t + \frac{\pi}{4}\right)$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}\pi} \approx 1.15s \implies T_s \leq 0.58s$$

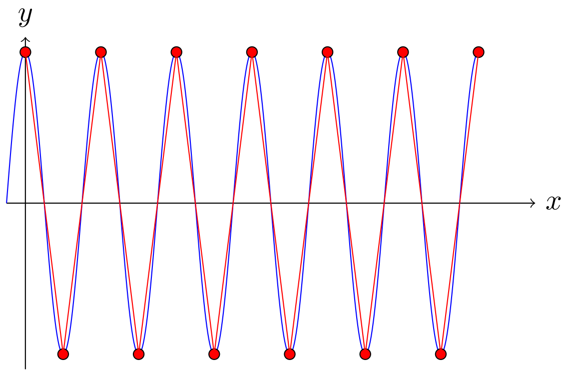
c) $x(t) = 2\sin(10\pi t)$, for this case draw the graphics and represent it.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = 0.2s \implies T_s \leq 0.1s$$

Depends on where we start our sampling, we may get different results.



Sampling starts at $t = 0s$.



Sampling starts at $t = 0.05s$.

In the first case, we get a DC reconstruction, which does not represent our signal. In the second case, we get a somewhat similar signal. If we sample somewhere between those phases, we will get a signal that has a similar frequency to our original signal, however amplitude information will be lost. We see that, in the case of our sampling frequency is twice the signal frequency, sampling qualtiy depends on the phase at which we start sampling our signal.

Problem 3

Simplify each expression to get it in the form $a + jb$.

a) $(1 + j3)(-3 - j1)$

$$(1 + j3)(-3 - j1) = -3 - j1 - j9 + 3 = 0 + j(-10)$$

b) $\frac{2-j4}{1-j3} - \frac{1-j4}{2-j3}$

$$\frac{2-j4}{1-j3} - \frac{1-j4}{2-j3} = \frac{2-j4}{1-j3} \cdot \frac{1+j3}{1+j3} - \frac{1-j4}{2-j3} \cdot \frac{2+j3}{2+j3} = \frac{1}{10}(14+j2) - \frac{1}{13}(14-j5) = \frac{21}{65} + j\frac{38}{65}$$

Problem 4

Simplify each expression to get it in the form $r \cdot e^{j\theta}$.

a) $\frac{1+j3}{-3-j1}$

$$\frac{1+j3}{-3-j1} = \frac{1+j3}{-3-j1} \cdot \frac{-3+j1}{-3+j1} = \frac{1}{10}(-6-j8) \overset{\text{3rd quadrant}}{=} \frac{\sqrt{6^2+8^2}}{10} e^{j\left(\pi+\arctan\left(\frac{-8}{-6}\right)\right)} \approx e^{j4.07}$$

b) $\frac{2-j4}{1-j3}$

$$\frac{2-j4}{1-j3} = \frac{2-j4}{1-j3} \cdot \frac{1+j3}{1+j3} = \frac{1}{10}(14-j2) \overset{\text{1st quadrant}}{=} \frac{\sqrt{14^2+2^2}}{10} e^{j\arctan\left(\frac{2}{14}\right)} \approx \sqrt{2}e^{j0.14}$$

Problem 5

Express the signal in terms of complex exponentials.

a) $x[n] = 2\cos\left(\frac{2\pi}{8}n\right) + j3\sin\left(\frac{2\pi}{8}n\right)$

$$x[n] = e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n} + \frac{3}{2}e^{j\frac{\pi}{4}n} - \frac{3}{2}e^{-j\frac{\pi}{4}n} = \frac{5}{2}e^{j\frac{\pi}{4}n} - \frac{1}{2}e^{-j\frac{\pi}{4}n}$$

b) $x[n] = \cos(\pi n)$

$$x[n] = \frac{e^{j\pi n} + e^{-j\pi n}}{2}$$

c) $x[n] = \cos\left(\frac{2\pi}{8}n + \frac{\pi}{3}\right)$

$$x[n] = \frac{e^{j\left(\frac{2\pi}{8}n+\frac{\pi}{3}\right)} + e^{-j\left(\frac{2\pi}{8}n+\frac{\pi}{3}\right)}}{2} = \frac{1}{2}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)e^{j\frac{\pi}{4}n} + \frac{1}{2}\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)e^{-j\frac{\pi}{4}n}$$

Problem 6

Consider a discrete signal $x[n] = 2 \cdot \sin\left[\frac{\pi}{5}n - \frac{\pi}{3}\right]$

a) List the values of $x[n]$ for $n = -3, -1, 0, 1, 3$.

The values are approximately, in order: $-0.416, -1.99, -1.73, -0.81, 1.49$.

b) Is $x[n]$ even-symmetric, odd-symmetric, or neither? If the signal is not symmetric extract its even and odd components.

The signal is neither even nor odd symmetric. The even and odd components are:

$$x_{\text{even}}[n] = \frac{x[n]+x[-n]}{2} = \sin\left[\frac{\pi}{5}n - \frac{\pi}{3}\right] - \sin\left[\frac{\pi}{5}n + \frac{\pi}{3}\right] = -\sqrt{3}\cos\left[\frac{\pi}{5}n\right]$$

$$x_{\text{odd}}[n] = \frac{x[n]-x[-n]}{2} = \sin\left[\frac{\pi}{5}n - \frac{\pi}{3}\right] + \sin\left[\frac{\pi}{5}n + \frac{\pi}{3}\right] = \sin\left[\frac{\pi}{5}n\right]$$

c) If the waveform is periodic, what is its period?

It is periodic and the period is $\frac{\pi}{5}n = 2\pi k$, and at $k = 1$, $n = 10$. Period is 10.

d) Is $x[n]$ an energy signal, a power signal, or neither? If it is an energy signal, find its energy. If it is a power signal, find its average power.

It is a power signal, as it has infinite energy and is constantly oscliating. The average power can be calcualted by either the infinite sum, or we simply know that average power of a sine/cosine wave is amplitude squared over two, which means average power $P_{\text{average}} = \frac{2^2}{2} = 2$.