Digital version

ENGS252 Signals and Systems - Homework 5

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Problem 1

Determine the sinusoid $c(n)$ in polar form, that is the sum of the pair of given sinusoids, $a(n)$ and $b(n)$, $c(n) = a(n) + b(n)$. Find the samples of the sinusoid $c(n)$ for $n = 0$ to $n = 5$ and verify that the samples are the same as the sum of the samples of $a(n)$ and $b(n)$, $a(n) + b(n)$.

$$a[n] = 4\cos\left[\frac{\pi}{4}n + \frac{\pi}{3}\right],\ b[n] = \cos\left[\frac{\pi}{4}n - \frac{\pi}{4}\right]$$

$$c[n] = 4\cos\left[\frac{\pi}{4}n + \frac{\pi}{3}\right] + \cos\left[\frac{\pi}{4}n - \frac{\pi}{4}\right] = 4\left(\frac{1}{2}\cos\frac{\pi n}{4} - \frac{\sqrt{3}}{2}\sin\frac{\pi n}{4}\right) + \left(\frac{\sqrt{2}}{2}\cos\frac{\pi n}{4} + \frac{\sqrt{2}}{2}\sin\frac{\pi n}{4}\right)$$

$$c[n] = \left(\frac{4+\sqrt{2}}{2}\right)\cos\frac{\pi n}{4} - \left(\frac{-4\sqrt{3}+\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2}\sin\frac{\pi n}{4} \approx 3.86\cos\left(\frac{\pi n}{4} + 0.79\right)$$

n	$a[n]$	$b[n]$	$a[n] + b[n]$	$c[n]$
0	2	0.71	2.71	2.72
1	-1.04	1	-0.04	-0.02
2	-3.46	0.71	-2.76	-2.74
3	-3.86	0	-3.86	-3.86
4	-2	-0.71	-2.71	$-2/72$
5	-1.04	1	-0.04	-0.02

Problem 2

What must be the time resolution of the ADC system to represent these signals? Determine if the following signal (with t expressed in seconds) is periodic. If yes, determine the fundamental period.

$$x(t) = 3\sin(4\pi t) + 7\sin(3\pi t)$$

The fastest angular frequency of the given signal is 4π, which gives a period of $0.5s$. If we use the theory, a minimum of $0.25s$ time resolution ADC is required to capture the signal, with the minimum recommended being $0.1s$ to get a better picture.

The signal is composed of two sine signals, one with 4π angular frequency, the other with 3π. Hence, their periods are $\frac{1}{2}s$ and $\frac{2}{3}s$. The smallest number that is divided by both numbers is $2s$, which is the period of our combined signal, and hence the signal is periodic.

Problem 3

Determine all generated signals in output of 16QAM (phases and amplitudes included). The following data is given for amplitude and phase states:

	bit state 0	bit state 1
Amplitude states	1V	3.73V
Phase states	π	0

We should consider positive and negative sine and cosine waves (with $\frac{\pi}{4}$ phases to begin with) with the following four cases. We get 16 different possible waves, which each correspond to a specific 4-bit symbol:

- $\sin\left(\omega_c t + \frac{\pi}{4}\right)$
- $3.73\sin\left(\omega_c t + \frac{5\pi}{4}\right)$
- $\sin\left(\omega_c t + \frac{\pi}{4}\right)$
- $-3.73\sin\left(\omega_c t + \frac{5\pi}{4}\right)$
- $-\sin\left(\omega_c t + \frac{\pi}{4}\right)$
- $-3.73\sin\left(\omega_c t + \frac{5\pi}{4}\right)$
- $-\sin\left(\omega_c t + \frac{\pi}{4}\right)$
- $3.73\sin\left(\omega_c t + \frac{5\pi}{4}\right)$
- $\cos\left(\omega_c t + \frac{\pi}{4}\right)$
- $3.73\cos\left(\omega_c t + \frac{5\pi}{4}\right)$
- $\cos\left(\omega_c t + \frac{\pi}{4}\right)$
- $-3.73\cos\left(\omega_c t + \frac{5\pi}{4}\right)$
- $-\cos\left(\omega_c t + \frac{\pi}{4}\right)$
- $-3.73\cos\left(\omega_c t + \frac{5\pi}{4}\right)$
- $-\cos\left(\omega_c t + \frac{\pi}{4}\right)$
- $-3.73\cos\left(\omega_c t + \frac{5\pi}{4}\right)$

Problem 4

Given the complex sinusoids $x_1[n] = A_1 e^{j(\omega n + \theta_1)}$ and $x_x[n] = A_x e^{j(\omega n + \theta_x)}$, find the complex sinusoid $x[n] = x_1[n] + x_2[n] = Ae^{j(\omega n + \theta)}$:

$$x_1[n] = -2e^{j\left(\frac{\pi}{3}n + \frac{\pi}{3}\right)},\ x_2[n] = 3e^{j\left(\frac{\pi}{3}n - \frac{\pi}{6}\right)}$$

$$x[n] = -2e^{j\left(\frac{\pi}{3}n + \frac{\pi}{3}\right)} + 3e^{j\left(\frac{\pi}{3}n - \frac{\pi}{6}\right)} = e^{j\left(\frac{\pi}{3}n\right)}\left(-2e^{j\left(\frac{\pi}{3}\right)} + 3e^{j\left(-\frac{\pi}{6}\right)}\right)$$

$$= e^{j\left(\frac{\pi}{3}n\right)}\left(-2\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + 3\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)\right) = e^{j\left(\frac{\pi}{3}n\right)}\left(\frac{-2 + 3\sqrt{3}}{2} + j\frac{-3 - 2\sqrt{3}}{2}\right)$$

$$= e^{j\left(\frac{\pi}{3}n\right)}\left(\sqrt{13}e^{j\arctan - \frac{24 + 13\sqrt{3}}{23}}\right) \approx 3.51e^{j\left(\frac{\pi}{3}n - 1.11\right)}$$

Problem 5

The transmission bitrate of the channel is 8Kb/s and the modulation is 16QAM calculate the baud rate of the transmission.

Since transmission is 8000 bits per second and 16QAM uses 4 bit symbols, the baud rate is 2000.