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ENGS252 Signals and Systems - Homework 7

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Problem 1

A mass of 0.2 kg is put on frictionless horizontal plane and is connected by a spring of k=10 N/m to a wall. The system is at equilibrium. It is given a speed of $v(0) = 40m/s$ to the mass, the direction of the velocity is away from the wall. Due to viscous friction that depends on the velocity as $F = -0.15v$ the initiated oscillations will damp away.

a) Using the derived equation for the damping oscillations find the parameters that depend on the intial conditions of the system.

$$0.2\ddot{x} + 0.15\dot{x} + 10x = 0 \implies r = \frac{-0.75 \pm \sqrt{0.75^2 - 4 \cdot 50}}{2}$$

Underdamped system, therefore the solution has the following form:

$$x(t) = e^{-\beta t}\left(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t)\right) = Ae^{-\beta t}\cos(\omega t + \varphi_0)$$

A and φ_0 depend on initial conditions.

$$x(0) = 0 \implies C_1 = 0,\ v(0) = \dot{x}(0) = 40 \implies 40 = C_2\omega_d,\ C_2 = \frac{40}{\omega_d} \approx 5.66$$

$$\therefore \varphi_0 = 0, A \approx 5.66$$

b) What is the frequency of oscillations?

$$\omega_d = \sqrt{\omega_0^2 - \beta^2} = \sqrt{50 - 0.375^2} \approx 7.06$$

c) How much time it will take to decrease the energy of the system twice.

Need to find the time it takes when the amplitude is $\frac{A}{\sqrt{2}}$.

$$e^{-\beta t} = \frac{1}{\sqrt{2}} \implies t = \frac{\ln\left(\sqrt{2}\right)}{\beta} \approx 1.85s$$

Problem 2

An inductor of $L = 1mH$, capacitor of capacitance $C = 1\mu F$, and a resistor of resistance of $R = 1k\Omega$ are connected in series. The capacitor initially is charged up to $16V$.

a) Write down the Kirchhoff's law for this circuit. Compare the yielded equation with the damping mechanical oscillations discussed during the lecture.

$$\frac{q(t)}{C} + L\dot{i}(t) + Ri(t) = 0 \implies \ddot{q}(t) + \frac{R}{L}\dot{q}(t) + \frac{1}{LC}q(t) = 0$$

$$\beta = \frac{R}{2L} = 5 \cdot 10^5.\ \omega^2 = \frac{1}{LC} \approx 3.16 \cdot 10^4$$

Overdamped system.

b) How the voltage on the capacitor will change in time? Write the expression of dependence of voltage on time. Hint: Use analogy with damping spring oscillations.

$$r = \frac{-10^6 \pm \sqrt{10^12 - 4 \cdot 10^9}}{2}$$

$$q(t) = Ae^{-r_1 t} + Be^{-r_2 t}$$

c) What is the resonance frequency of this system?

$$\omega_0 = \sqrt{\frac{1}{LC}} \approx 3.16 \cdot 10^4 \implies f_0 = \frac{\omega_0}{2\pi} = \frac{3.16 \cdot 10^4}{2\pi} \approx 5030Hz$$

Problem 3

For a continuous-time system with input x(t) and output y(t) governed by the differential equation

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 7\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 12y(t) = x(t)$$

for $t \geq 0^+$

a) What are the characteristic roots of the differential equation?

$$r^2 + 7r + 12 = 0 \implies r = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 12}}{2} = \frac{-7 \pm 1}{2} \implies \begin{cases} r_1 = -4 \\ r_2 = -3 \end{cases}$$

b) Find the zero-input response assuming non-zero initial conditions.

Denote initial conditions $y(0) = y_0$ and $y'(0) = y_1$.

$$y(t) = Ae^{-3t} + Be^{-4t} \implies y'(t) = -3Ae^{-3t} - 4Be^{-4t} \implies \begin{cases} y_0 = A + B \\ y_1 = -3A - 4B \end{cases}$$

$$\begin{cases} A = 4y_0 + y_1 \\ B = -3y_0 - y_0 \end{cases} \implies y(t) = (4y_0 + y_1)e^{-3t} + (-3y_0 - y_1)e^{-4t}$$