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ENGS252 Signals and Systems - Homework 8

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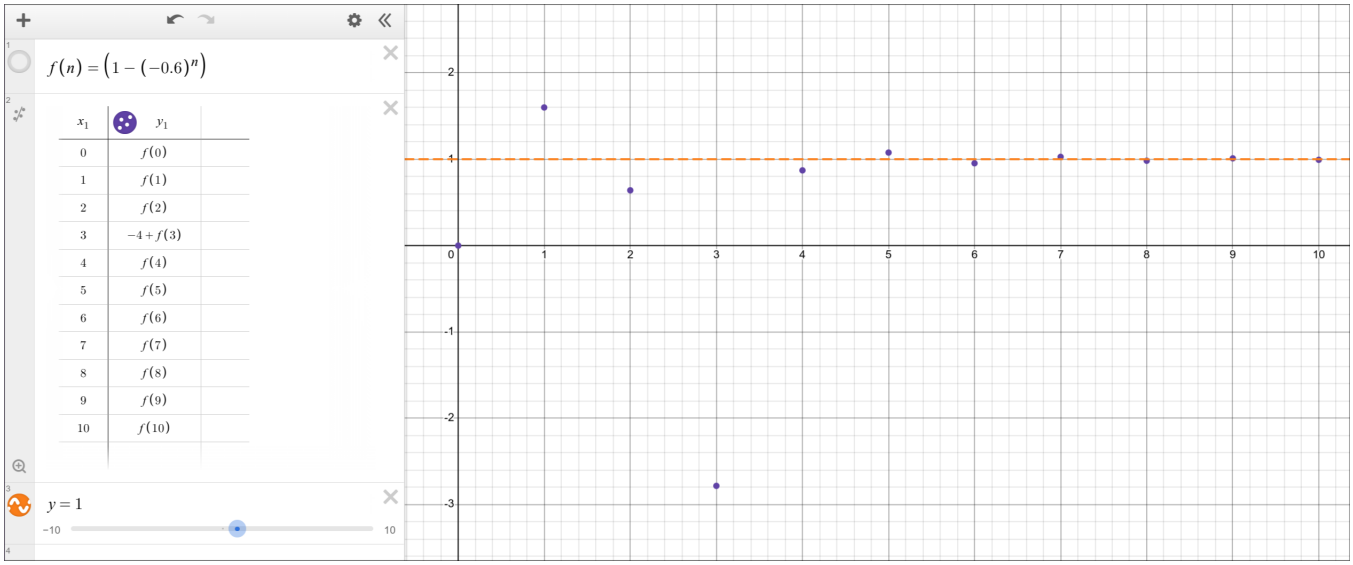
Problem 1

Find the steady-state response of the system, with the impulse response:

$$h(n) = -4\delta(n-3) + (1-(-0.6)^n)u(n),\ n = 0, 1, 2, ...$$

Draw the graph of $h[n]$ vs n.

At the steady state, the system response goes to 1, as the impulse is only present at the point $n = 3$, and the $(-0.6)^n$ term approaches zero, for large values of n. Because of the unit step function, the system is zero for $n < 0$.



Problem 2

Is the system governed by the given difference equation, with input $x[n]$ and output $y[n]$, linear? Let $\{x_1(n), n = 0, 1, 2, 3\} = \{1, 2, 3, 2\}, \{x_2(n), n = 0, 1, 2, 3\} = \{2, 3, 0, 4\}$ and $x(n) = 2x_1(n) - 3x_2(n)$. Assuming that the initial condition $y(-1)$ is zero, compute the first four output values and verify the conclusion.

Compute the values of $x(n)$: $\{x(n),\ n = 0, 1, 2, 3\} = \{-4, -5, 6, -8\}$.

a) $y(n) = x(n) + x(n)y(n-1)$

$$\begin{cases} y_1(n) = x_1(n) + x_1(n)y_1(n-1) \\ y_2(n) = x_2(n) + x_2(n)y_2(n-1) \end{cases}$$

$y(n) = 2y_1(n) - 3y_2(n) = 2x_1(n) - 3x_2(n) + 2x_1(n)y_1(n-1) - 3x_2(n)y_2(n-1) \neq x(n) + x(n)y(n-1)$

System is not linear, as it does not hold the superposition principle.
$\{y_1(n),\ n = 0,1,2,3\} = \{1,4,15,32\},\ \{y_2(n),\ n = 0,1,2,3\} = \{2,9,0,4\},\ \{2y_1(n) - 3y_2(n),\ n = 0,1,2,3\} = \{-4,-16,30,52\} \neq \{y_3(n),\ n = 0,1,2,3\} = \{-4,15,96,-776\}$

b) $y(n) = nx(n) + y(n-1) - 3\cos\pi$
$y(n) = nx(n) + y(n-1) + 3$, which is clearly non-linear because if $x(0) = 0,\ y(0) = 3 \neq 0$ (will break the superposition principle). $\{y_1(n),\ n = 0,1,2,3\} = \{3,8,17,26\},\ \{y_2(n),\ n = 0,1,2,3\} = \{3,9,12,27\},\ \{2y_1(n) - 3y_2(n),\ n = 0,1,2,3\} = \{-3,-11,-2,-29\} \neq \{y_3(n),\ n = 0,1,2,3\} = \{3,1,16,-5\}$

Problem 3

Is the system governed by the given difference equation, with input $x(n)$ and output $y(n)$, time-invariant? Let $\{x(n), n = 0,1,2,3,4,5,6,7,8\} = \{2,1,3,3,4,2,5,1,3\}$. Assuming that the initial condition is zero, compute the first four output values and verify the conclusion to the input $\{x(n-2), n = 2,3,4,5,6,7,8,9,10\} = \{2,1,3,3,4,2,5,1,3\}$

a) $y(n) = 2x(n) - \sin\left(\frac{\pi n}{2}\right) y(n-1)$

The system is time variant, because of the term $\sin\left(\frac{\pi n}{2}\right)$.
$\{y(n),\ n = 0,1,2,3\} = \{4,-2,6,12\} \neq \{y(n-2),\ n = 2,3,4,5\} = \{4,6,6,0\}$

b) $x(n) = x(n) + ny(n-1)$

The system is time variant, because of the term $ny(n-1)$, specifically because of the n.
$\{y(n),\ n = 0,1,2,3\} = \{2,3,9,30\} \neq \{y(n-2),\ n = 2,3,4,5\} = \{2,7,31,158\}$

c) $y(n) = 3x(n) - 2y(n-1) + 1$

The system is time invariant.
$\{y(n),\ n = 0,1,2,3\} = \{7,-10,30,-50\} = \{y(n-2),\ n = 2,3,4,5\} = \{7,-10,30,-50\}$

Problem 4

Is the system governed by the given difference equation causal system? Also mention if it is a recursive system?

a) $y(n) = x(n+1) + 3x(n-1) - y(n-1)$

Not casual system, as the term $x(n+1)$ depends on a future input. The system is recursive from the $y(n-1)$ term.

b) $y(n) = x(n) + ny(n-1)$

Casual system, as nothing depends on a future input. The system is recursive from the $y(n-1)$ term.

c) $y(n) = 3x(n) - 2x(n-1) + 1$

Casual system, as nothing depends on a future input. The system is not recursive, because no $y(n-a),\ a \in \mathbb{N}$ term.

Problem 5

The impulse response of a LTI system is given. Is the system stable?

a) $h(0) = 0,\ h(n) = \frac{1}{2^n},\ n = 1, 2, ...$

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1 \implies \text{system is stable}$$

b) $h(0) = 0,\ h(n) = \frac{1}{n^2},\ n = 1, 2, ...$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \text{ converges, as power is more than 1} \implies \text{system is stable}$$