Digital version

ENGS252 Signals and Systems - Homework 9

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Problem 1

Write code for constructing the circulant matrix that can be used for implementing the convolution.

The circulant matrix has the size $(n + m - 1) \times m$, where n is the size of the impulse response and m is the size of our input signal. Same definitions used in the pyhton code.

$$\begin{bmatrix} h(0) & 0 & ... & 0 \\ h(1) & h(0) & ... & 0 \\ ... & ... & ... & ... \\ h(n) & h(n-1) & ... & h(n-m) \\ 0 & 0 & ... & h(0) \end{bmatrix}$$

```
import numpy as np

def construct_matrix(h, m):
    n = len(h)
    matrix = np.zeros((n + m - 1, m))
    for i in range(n + m - 1):
        for j in range(m):
            if (i >= j and i - j < n):
                matrix[i][j] = h[i - j]
    return matrix
```

Sample output from construct_matrix([1, 2, 3, 4, 5, 6], 4):

```
[[1. 0. 0. 0.]
 [2. 1. 0. 0.]
 [3. 2. 1. 0.]
 [4. 3. 2. 1.]
 [5. 4. 3. 2.]
 [6. 5. 4. 3.]
 [0. 6. 5. 4.]
 [0. 0. 6. 5.]
 [0. 0. 0. 6.]]
```

Problem 2

Is the system governed by the given difference equation is a IIR or FIR system?

a) $y(n) = x(n) - 2x(n-1) - \frac{1}{2}y(n-1)$

Let's give impulse to this system and see the response:

$$y(0) = 1,\ y(1) = -2.5,\ y(2) = 1.25,\ y(3) = -0.625...$$

This is an IIR, as the current output depends on previous outputs, and response still continues after two steps (we have x[n] and x[n-1], hence for a FIR we should have zeros after the first two outputs).

b) $y(n) = x(n) + x(n-1) - x(n-2)$

This is a FIR system, as the current state depends only on current and past inputs (not outputs).

Problem 3

If the impulse response of the system is $h[n]$. Find the closed-form expression for the output of the system using the convolution method. The input signal is $x[n]$. List the values of the convolution output at $n = 0, 1, 2, 3, 4, 5$. For each case mention if the system is bounded system.

a) $\{x(n),\ n = 1, 2, 3, 4\} = \{3, 2, 4, 1\}$ and $h(n) = \delta(n)$

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{+\infty} x[m]h[n-m] = \sum_{m=-\infty}^{+\infty} x[m]\delta[n-m] = x[n]$$

$$\{y(n),\ n = 0, 1, 2, 3, 4, 5\} = \{3, 2, 4, 1, 0, 0\}$$

$$|x[n]| \leq 4 \text{ and } \sum_{m=0}^{\infty} |h[m]| = 1 < \infty \implies \text{bound system}$$

b) $x(n) = (0.5)^n u(n-2)$ and $h(n) = (0.7)^n u(n-1)$

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{+\infty} x[m]h[n-m] = \sum_{m=-\infty}^{+\infty} (0.5)^m u(m-2) \cdot (0.7)^{n-m} u(n-m-1)$$

$$\{y(n),\ n = 0, 1, 2, 3, 4, 5\} = \{0, 0, 0, 0.175, 0.21, 0.19075\}$$

$$y[n] = \sum_{m=2}^{n-1} (0.5)^m \cdot (0.7)^{n-m} = (0.7)^n \sum_{m=2}^{n-1} \left(\frac{5}{7}\right)^m$$

$$|x[n]| \leq 0.25 \text{ and } \sum_{m=0}^{\infty} |h[m]| = \sum_{m=1}^{\infty} 0.7^m = \sum_{m=0}^{\infty} (0.7^m) - 1 = \frac{1}{1-0.7} - 1 = \frac{7}{3} < \infty \implies \text{bound system}$$

c) $x(n) = e^{\frac{j2\pi}{6}n}$ and $h(n) = \delta(n-6)$

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{+\infty} x[m]h[n-m] = \sum_{m=-\infty}^{+\infty} x[m]\delta[n-m-6] = e^{\frac{j2\pi}{6}(n-6)} = e^{\frac{j2\pi}{6}n - j2\pi} = x[n]$$

$$\{y(n),\ n = 0,1,2,3,4,5\} = \{1, e^{\frac{j\pi}{3}}, e^{\frac{j2\pi}{3}}, e^{\frac{j3\pi}{3}}, e^{\frac{j4\pi}{3}}, e^{\frac{j5\pi}{3}}\}$$

$$|x[n]| = 1 \text{ and } \sum_{m=0}^{\infty} |h[m]| = 1 < \infty \implies \text{bound system}$$

Problem 4

Derive the closed-form expression for the complete response (by finding the zero-state response using the convolution-summation and the zero-input response) of the system governed by the difference equation

$$y(n) = 2x(n) - x(n-1) + \frac{1}{3}y(n-1)$$

with the initial condition $y(-1) = 2$ and the input $x(n) = u(n)$, the unit-step function. List the values of the complete response $y(n)$ at $n = 0,1,2,3,4,5$. Deduce the expressions for the transient and steady-state responses of the system.

$$x[n] = u[n] \implies y[n] = 2u[n] - u[n-1] + \frac{1}{3}y[n-1]$$

$$\{y(n),\ n = 0,1,2,3,4,5\} = \left\{\frac{8}{3}, \frac{17}{9}, \frac{44}{27}, \frac{125}{81}, \frac{368}{243}, \frac{1097}{729}\right\}$$

We see a pattern of constant plus something times $\frac{1}{3}^n$, therefore $y[n] = \frac{3}{2} + \frac{7}{6} \cdot \left(\frac{1}{3}\right)^n$. The steady state part is $\frac{3}{2}$, and the transient part is the $\left(\frac{1}{3}\right)^n$.