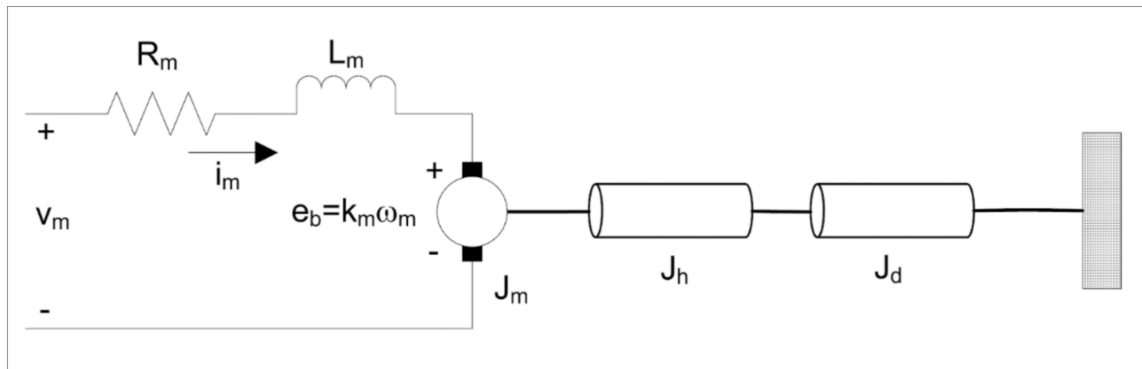




ENGS261 Control Systems - Homework 1

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September 15, 2026



Task 1: Transfer-function derivation

Using the motor diagram, the parameter table, and the modeling procedure in Nise, Section 2.8:

- Draw the electrical and mechanical parts of the model and define the direction and sign convention for voltage, current, torque, speed, and position.
- Write the electrical, motor torque/back-EMF, mechanical, and speed-position relations needed for this system. State where the neglected inductance assumption is used.

We write the Kirchhoffs second law for the electrical circuit:

$$v_m(t) = R i_m(t) + L \frac{di_m}{dt} + k_m \omega_m$$

We write the relation between torque and angular acceleration:

$$\tau = J \alpha \implies J_{eq} \frac{d\omega_m}{dt} = k_t i_m(t)$$

- Eliminate the internal current and torque variables to obtain one differential equation relating $v_m(t)$ and $\omega_m(t)$.

$$i_m(t) = \frac{J_{eq}}{k_t} \frac{d\omega_m}{dt} \implies v_m(t) = R \frac{J_{eq}}{k_t} \frac{d\omega_m}{dt} + L \frac{J_{eq}}{k_t} \frac{d^2\omega_m}{dt^2} + k_m \omega_m$$

d) Take the Laplace transform using zero initial conditions and derive

$$G_\omega(s) = \frac{\Omega_m(s)}{V_m(s)}$$

After taking the laplace transform with zero initial conditions, we get:

$$V_m(s) = \frac{RJ_{eq}}{k_t}s\Omega_m(s) + \frac{LJ_{eq}}{k_t}s^2\Omega_m(s) + k_m\Omega_m(s)$$

$$G_\omega(s) = \frac{\Omega_m(s)}{V_m(s)} = \frac{1}{\frac{RJ_{eq}}{k_t}s + \frac{LJ_{eq}}{k_t}s^2 + k_m} = \frac{k_t}{LJ_{eq}s^2 + RJ_{eq}s + k_tk_m}$$

We can greatly simplify the equation, if we neglect the inductance:

$$G_\omega(s) = \frac{k_t}{RJ_{eq}s + k_tk_m}$$

e) Use the relationship between angular speed and angular position to derive

$$G_\theta(s) = \frac{\Theta_m(s)}{V_m(s)}$$

We know that, $\omega_{\square\square\square}(t) = \frac{d\theta_m}{dt}$, therefore with zero initial conditions: $\Omega_m(s) = s\Theta(s)$, hence:

$$G_\theta(s) = \frac{1}{s} \cdot \frac{k_t}{RJ_{eq}s + k_tk_m}$$

f) Write $G_\omega(s)$ in the standard form:

$$G_\omega(s) = \frac{K}{\tau s + 1}$$

In the standard form, we get the following equation, with $K = \frac{1}{k_m}$ and $\tau = \frac{RJ_{eq}}{k_tk_m}$:

$$G_\omega(s) = \frac{\frac{1}{k_m}}{\frac{RJ_{eq}}{k_tk_m}s + 1}$$

Task 2: Numerical model

Use the parameter table and the baseline assumption $b = 0$.

a) Calculate J_d and J_{eq} .

$$J_d = \frac{1}{2} \cdot 0.053 \cdot 0.0248^2 \approx 1.630 \cdot 10^{-5} \text{ kgm}^2$$

$$J_{eq} = 1.4 \cdot 10^{-6} + 0.6 \cdot 10^{-6} + 1.630 \cdot 10^{-5} \approx 1.83 \cdot 10^{-5} \text{ kgm}^2$$

b) Calculate the numerical values of K and τ , including units.

$$K = \frac{1}{0.0422} \approx 23.70 \text{ rad/sV}$$

$$\tau = \frac{7.5 \cdot 1.83 \cdot 10^{-5}}{0.0422 \cdot 0.0422} \approx 0.0771 \text{ } \Omega\text{kgm}^2\text{Arad/sNmV}$$

- c) Write the numerical voltage-to-speed and voltage-to-position transfer functions.

$$G_\omega(s) \approx \frac{23.70}{0.0771s + 1}$$

$$G_\theta(s) \approx \frac{23.70}{0.0771s^2 + s}$$

- d) Without recalculating, explain how a positive value of b would affect K and τ .

Damping would make the response slower, so a lower value of K and higher value of τ .

Task 3: Analytical step response

Apply the voltage input

$$v_m(t) = A_v u(t), \quad A_v = 1 \text{ V}$$

with zero initial position and zero initial speed.

- a) Find $V_m(s)$, then calculate $\Omega_m(s) = G_\omega(s)V_m(s)$.

$$V_m(s) = \frac{1}{s}$$

$$\Omega_m(s) = \frac{23.70}{s(0.0771s + 1)}$$

- b) Use partial fractions and the inverse Laplace transform to derive $\omega_m(t)$.

$$\Omega_m(s) = \frac{23.70}{s(0.0771s + 1)} = \frac{23.70}{s} - \frac{1.82727}{0.0771s + 1} = \frac{23.70}{s} - \frac{23.7}{s + 12.97}$$

$$\omega_m(t) = 23.7 - 23.7e^{-12.97t} = 23.7(1 - e^{-12.97t})$$

- c) Derive $\theta_m(t)$ either from $G_\theta(s)$ or by integrating $\omega_m(t)$.

$$\theta_m(t) = 23.7t + 1.83e^{-12.97t}$$

- d) Determine the steady-state speed and evaluate the speed at $t = \tau$.

Steady state speed is 23.7 rad/s, speed at $t = \tau$ is $23.7(1 - e^{-1}) \approx 14.5 \text{ rad/s}$.

- e) Explain why the speed can approach a constant while the position continues to increase.

Trivial high school level question. Speed is change of position. Constant speed means position changes at a constant rate.