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# ENGS121 Mechanics Lab Section B
# Testing the equivalence of the six faces of a die

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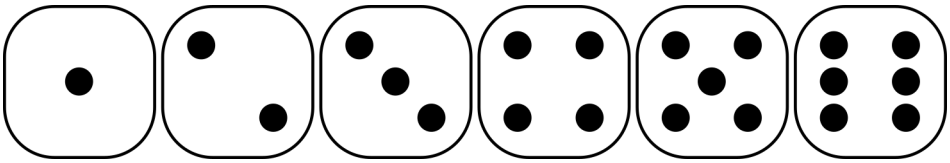
## 1 Introduction

A regular die has six faces, each with a number of holes ranging from one to six, representing six possible cases. The null hypothesis is that the probability of each case occurring after a throw is assumed to be 1/6, because after each throw a die shows one of the six cases, and they are assumed to be equally probable. The alternative hypothesis is that the holes in the die shift the center of mass from the center of the cube, resulting in an unequal probability for each case to occur.



An experiment was carried out to test the hypothesis using the Chi Square test at the 5% significance level.

## 2 Measurements and data

A group of 10 dice were tossed 20 times and a box was used to contain the dice from falling off the table. As a die has six faces, we assign six variables to represent the number of times that case (”1”, ”2”, ”3”, ”4”, ”5” or ”6”) has occurred in that toss. The results for each of the tosses were record (Table 1).

Errors may be caused from the manufacturing process of the die, because no factory can produce dice that are identical and have equal probability for each of the six faces to result. Another error can be caused when a die is thrown from a short height and the die does not have enough energy to move chaotically, so the result is dependent from the initial position. To reduce this error, the dice were thrown from a height that ensures random results.

| **toss** | "1" | "2" | "3" | "4" | "5" | "6" | **toss** | "1" | "2" | "3" | "4" | "5" | "6" |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 3 | 2 | 3 | 0 | 1 | 11 | 2 | 0 | 0 | 3 | 2 | 1 |
| 2 | 0 | 2 | 2 | 2 | 2 | 2 | 12 | 2 | 2 | 2 | 2 | 1 | 1 |
| 3 | 1 | 2 | 1 | 2 | 3 | 1 | 13 | 2 | 4 | 4 | 0 | 0 | 2 |
| 4 | 1 | 2 | 0 | 1 | 2 | 4 | 14 | 0 | 4 | 4 | 1 | 2 | 3 |
| 5 | 0 | 3 | 2 | 2 | 2 | 1 | 15 | 1 | 1 | 1 | 1 | 2 | 3 |
| 6 | 2 | 2 | 1 | 2 | 0 | 3 | 16 | 2 | 1 | 1 | 2 | 1 | 1 |
| 7 | 4 | 1 | 0 | 1 | 3 | 1 | 17 | 0 | 1 | 1 | 1 | 6 | 1 |
| 8 | 0 | 0 | 2 | 5 | 0 | 3 | 18 | 2 | 0 | 0 | 2 | 2 | 1 |
| 9 | 1 | 3 | 3 | 0 | 1 | 2 | 19 | 0 | 2 | 2 | 2 | 1 | 3 |
| 10 | 1 | 2 | 2 | 2 | 2 | 1 | 20 | 1 | 2 | 2 | 2 | 1 | 4 |

Table 1: Frequencies of each case from the experiment

# 3 Calculations and plots

The degree of freedom for this experiment is 5, as by knowing the results of the five cases, we can calculate the result of the remaining one case, by subtracting them from the total number of throws per toss, which is ten.

The probabilities were calculated by dividing the frequency of each case by the number of total throws.

The expected and occurred probabilities and frequencies were calculated for 5 tosses (Table 2), 10 tosses (Table 3) and 20 tosses(Table 4), frequency distribution (Figure 1) and probability distribution (Figure 2) were graphed.

| Cases | Observed Freq. (O) | Observed Prob. | Expected Prob. | Expected Freq. (E) | $\frac{(O-E)^2}{E}$ |
|---|---|---|---|---|---|
| "1" | 3 | 0.060 | 0.167 | 8.333 | 3.413 |
| "2" | 12 | 0.240 | 0.167 | 8.333 | 1.613 |
| "3" | 7 | 0.140 | 0.167 | 8.333 | 0.213 |
| "4" | 10 | 0.200 | 0.167 | 8.333 | 0.333 |
| "5" | 9 | 0.180 | 0.167 | 8.333 | 0.053 |
| "6" | 9 | 0.180 | 0.167 | 8.333 | 0.053 |
| $\sum$ | 50 | 1 | 1 | 50 | 5.680 |

Table 2: Calculations for 5 tosses (50 dice throws)

| Cases | Observed Freq. (O) | Observed Prob. | Expected Prob. | Expected Freq. (E) | $\frac{(O-E)^2}{E}$ |
|---|---|---|---|---|---|
| "1" | 11 | 0.110 | 0.167 | 16.667 | 1.927 |
| "2" | 20 | 0.200 | 0.167 | 16.667 | 0.667 |
| "3" | 25 | 0.150 | 0.167 | 16.667 | 0.167 |
| "4" | 20 | 0.200 | 0.167 | 16.667 | 0.667 |
| "5" | 25 | 0.150 | 0.167 | 16.667 | 0.167 |
| "6" | 19 | 0.190 | 0.167 | 16.667 | 0.327 |
| $\sum$ | 100 | 1 | 1 | 100 | 3.920 |

Table 3: Calculations for 10 tosses (100 dice throws)

| Cases | Observed Freq. (O) | Observed Prob. | Expected Prob. | Expected Freq. (E) | $\frac{(O-E)^2}{E}$ |
|---|---|---|---|---|---|
| "1" | 23 | 0.115 | 0.167 | 33.333 | 3.203 |
| "2" | 37 | 0.185 | 0.167 | 33.333 | 0.403 |
| "3" | 32 | 0.160 | 0.167 | 33.333 | 0.053 |
| "4" | 36 | 0.180 | 0.167 | 33.333 | 0.213 |
| "5" | 33 | 0.165 | 0.167 | 33.333 | 0.003 |
| "6" | 39 | 0.195 | 0.167 | 33.333 | 0.963 |
| $\sum$ | 200 | 1 | 1 | 200 | 4.840 |

Table 4: Calculations for 20 tosses (200 dice throws)

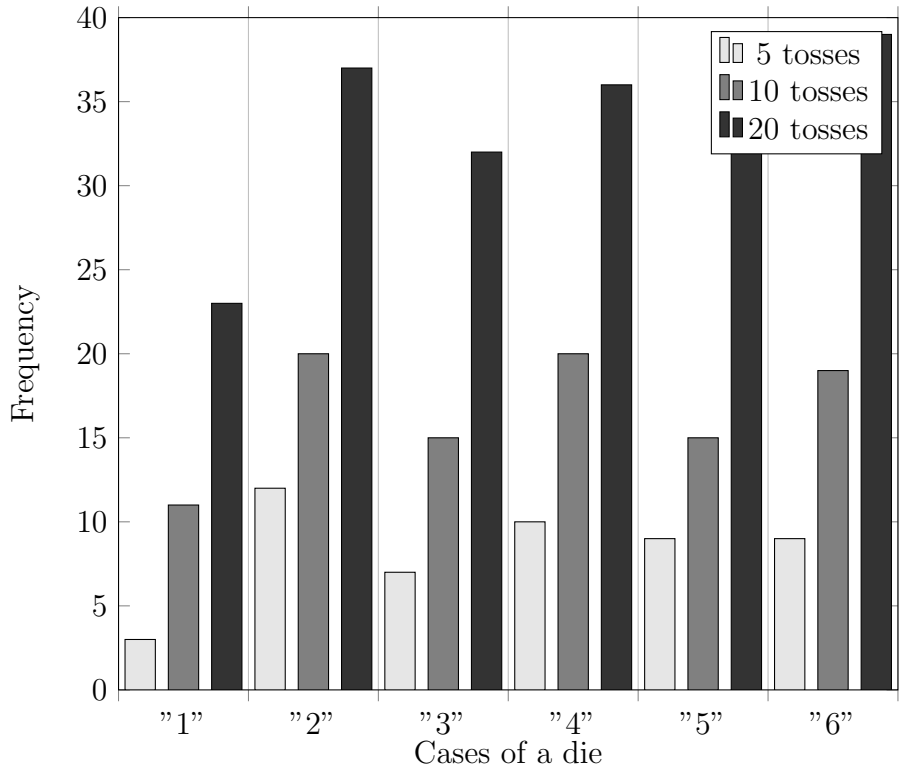



Figure 1: Frequency distribution for 5, 10, 20 tosses (50, 100, 200 dice throws)

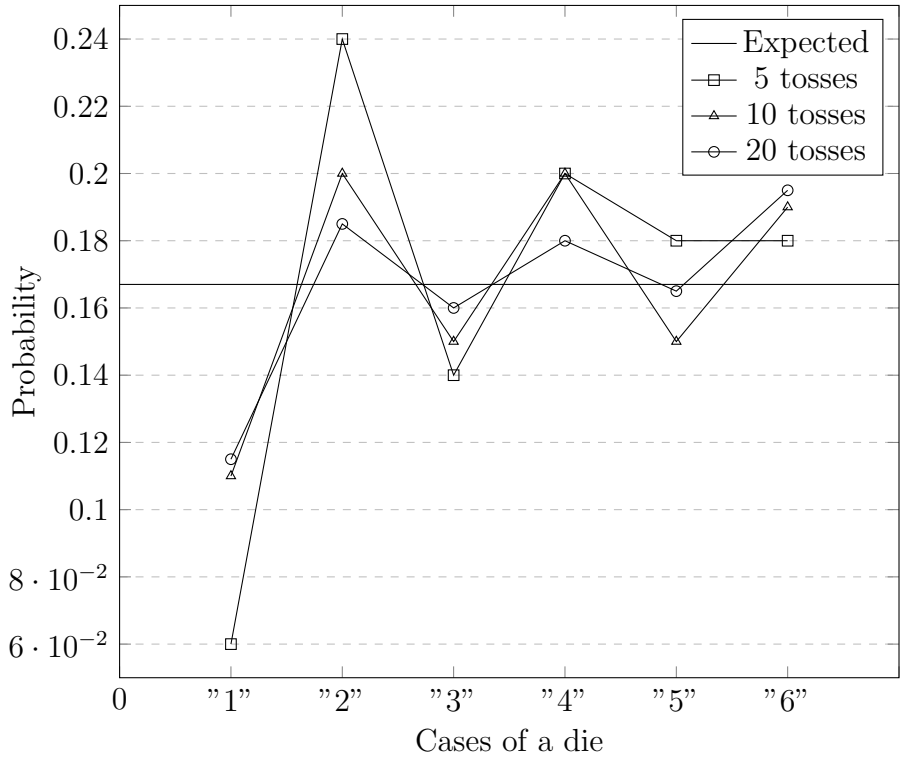



Figure 2: Probability distribution for 5, 10, 20 tosses (50, 100, 200 dice throws)

The $\chi^2$ values for 5, 10 and 20 tosses are calculated to be 5.680, 3.920 and 4.840 respectively. From the Chi Square table for $df = 5$ we get a critical value of $\chi^2_{0.05} = 16.750$.

# 4 Evaluation

We did not get much information from the Frequency distribution histogram, because it is difficult to compare values from different data sets. The probability distribution however allows us to compare the results of the three data sets that we chose. While the data for 5 tosses is far from the expected value, the data for 10 and 20 tosses are more closer to the assumed probability of 1/6.

Because the experimental value of $\chi^2 = 4.840$ is lower the critical value of $\chi^2_{0.05} = 16.750$, the alternative hypothesis is not supported and the null hypothesis is plausible, given the data, at the 5% significance level.

# 5 Conclusion

An experiment was carried out to test the equivalence of the six faces of a die at the 5% significance level. After analyzing the experimental results, it is concluded that the six faces of a die are equivalent, the probability for each face to occur is 1/6 and the holes in the faces have an insignificant effect on this probability.

# References

Kurghinyan, B. (2024, Feb). *Chi_square_test: Histograms for discrete data.*

Turney, S. (2023, Jun). *Chi-square (2) tests: Types, formula examples.* Retrieved from `https://www.scribbr.com/statistics/chi-square-tests/`