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ENGS121 Mechanics Lab Section B
Testing the uniformity of the number of crossing in Buffon's Needle experiment.

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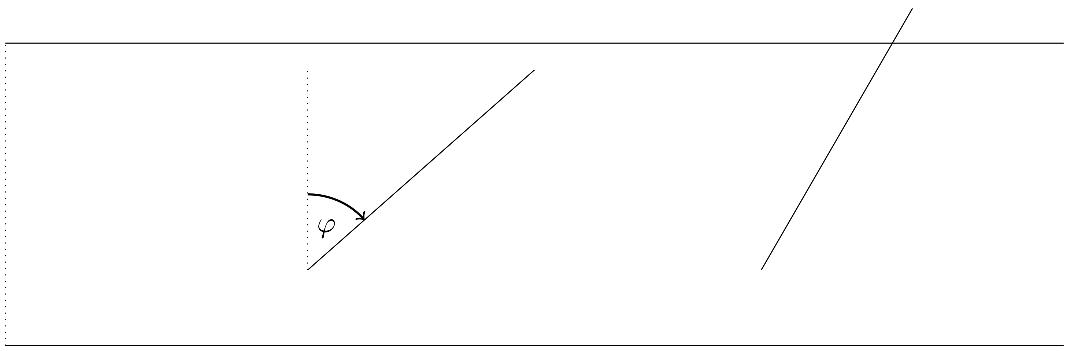
1 Introduction

This experiment investigates the behavior of random numerical data, the number of crossing in the Buffon's needle experiment. The experiment consists of dropping same length toothpicks on horizontal lines with the same distance between each other and finding patterns between throws.



To calculate the probability of a crossing theoretically, we can find the probabilities for all angles of the needle and add them. The length of the needle and the gap between the lines are considered 1. If a needle has angle φ, the projection of the needle on the vertical axis is $|\sin\varphi|$. In cases there is a crossing, the tip of the needle has to be between 0 and $|\sin\varphi|$ away from one of the lines and the maximum distance is 1. Hence, the probability of a crossing is $P_1 = \frac{|\sin\varphi|}{1} = |\sin\varphi|$. Now we consider the values of φ. There is a probability of $P_2 = \frac{\Delta\varphi}{2\pi}$ for getting an angle in the range $\Delta\varphi$. Now we multiply the two probabilities and sum them from $0 \leq \varphi \leq 2\pi$ for small values of $\Delta\varphi$, which can be done with integration.

$$\int_0^{2\pi} |\sin\varphi| \cdot \frac{d\varphi}{2\pi} = \frac{1}{2\pi}\int_0^{\pi} 2\sin\varphi d\varphi = \frac{1}{\pi}\left[-\cos\varphi\right]_0^{\pi} = \frac{2}{\pi}$$

The null hypothesis states that the probability for a crossing is $\frac{2}{\pi}$, while there is an alternative hypotheses that states that the probabilities for a number of crossing is uniformly distributed among tosses.

An experiment was carried out to test the uniformity of the number of crossings in a toss and to find the probability of a toothpicks crossing a line, after a throw, at the 5% significance level.

2 Measurements and data

A length of toothpick was measured and a series of horizontal lines were printed with the same distance between them as the length measured. The paper was placed on a horizontal surface, to prevent toothpicks from rolling of the table and getting wrong results. A group of 10 toothpicks were tossed 40 times on the paper and the number of crossing for each toss was recorded. The variable is the number of crossings (a toothpick touching a horizontal line) per toss.

Errors can be caused from imperfect manufacturing of toothpicks and biased tosses. The toothpicks' length differences are insignificant, taking into account the 5% significance level we work with. To prevent the toothpicks from falling in groups, the toothpicks were dropped individually, one after the other, which resulted in more randomness, and the systematic error was reduced.

toss	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
crossings	9	6	7	9	6	5	6	5	6	5
toss	#11	#12	#13	#14	#15	#16	#17	#18	#19	#20
crossings	8	5	6	3	9	5	4	8	8	7
toss	#21	#22	#23	#24	#25	#26	#27	#28	#29	#30
crossings	9	3	9	3	5	6	5	2	4	5
toss	#31	#32	#33	#34	#35	#36	#37	#38	#39	#40
crossings	7	4	8	6	5	7	5	7	7	4

Table 1: Number of crossings per toss

3 Calculations and plots

toss	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
crossings	81	36	49	81	36	25	36	25	36	25
toss	#11	#12	#13	#14	#15	#16	#17	#18	#19	#20
crossings	64	25	36	9	81	25	16	64	64	49
toss	#21	#22	#23	#24	#25	#26	#27	#28	#29	#30
crossings	81	9	81	9	25	36	25	4	16	25
toss	#31	#32	#33	#34	#35	#36	#37	#38	#39	#40
crossings	49	16	64	36	25	49	25	49	49	16

Table 2: Number of crossings squared per toss

There are 11 possible cases for the number of crossings per toss, from 0 to 10. The degree of freedom for this experiment is 10.

Cases	Obs. Freq. (O)	Obs. Prob.	Exp. Prob.	Exp. Freq. (E)	$\frac{(O-E)^2}{E}$
0	0	0.000	0.091	3.640	3.640
1	0	0.000	0.091	3.640	3.640
2	1	0.025	0.091	3.640	1.915
3	3	0.075	0.091	3.640	0.113
4	4	0.100	0.091	3.640	0.036
5	10	0.250	0.091	3.640	11.113
6	7	0.175	0.091	3.640	3.102
7	6	0.150	0.091	3.640	1.530
8	4	0.100	0.091	3.640	0.036
9	5	0.125	0.091	3.640	0.508
10	0	0.000	0.091	3.640	3.640
$\sum$	40	1	1	40	29.271

Table 3: Frequency and probability calculations for crossings per toss

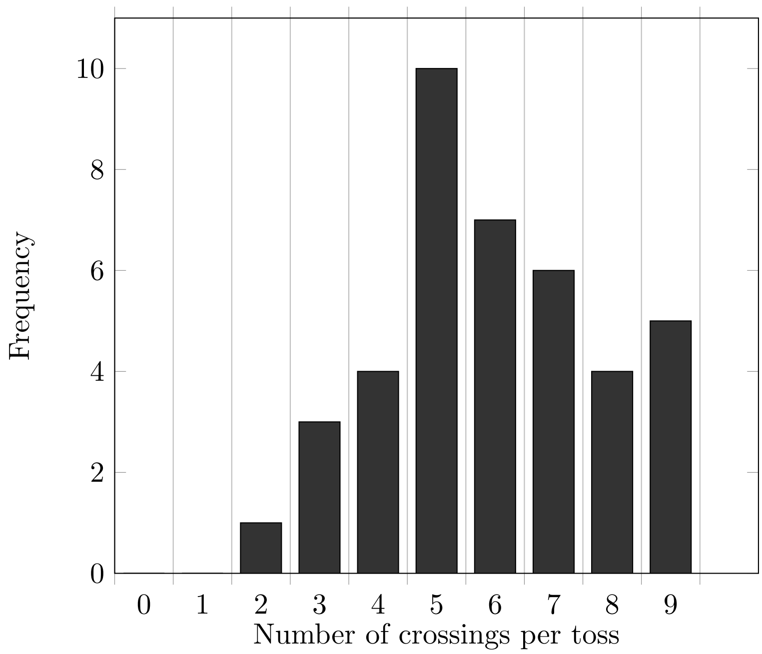


Figure 1: Frequency distribution

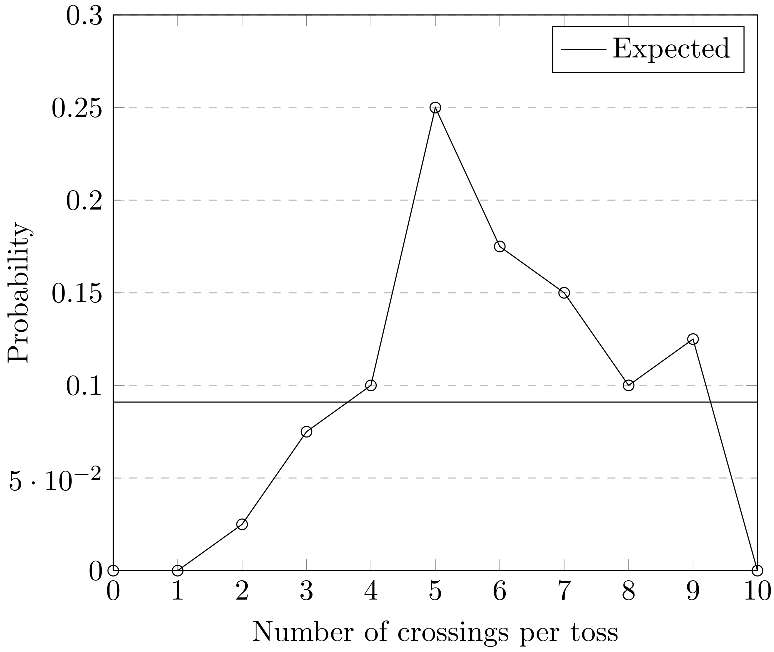


Figure 2: Probability distribution

In both frequency and probability distribution plots it is shown that the number of crossings per toss is not uniformly distributed, and there is a peak at value 5, which slowly decreases as we move further from 5. The χ^2 value is 29.271. From the Chi Square table for $df = 10$ we get a critical value of $\chi^2_{0.05} = 18.307$.

The total number of throws is 400 and the total number of crossings is 238, hence we can estimate by to be $\pi \approx 2 \cdot \frac{400}{238} = 3.36$.

Calculations show that $x_{mean} = 5.95$, $x^2_{mean} = 38.8$ and $\sigma = \sqrt{(x^2)_{mean} - (x_{mean})^2} = 1.84$, from values of Table 1 and Table 2, which translates to a mean probability of $p_{mean} = 0.595 \pm 0.184$.

4 Evaluation

With the χ^2 test, $\chi^2 = 29.271$, which is over the critical value of 18.307 at 5% significance level with 10 degree of freedom. Hence the alternative hypothesis is not supported and the null hypothesis is plausible, given the data from the experiment.

The value of π was approximately calculated to be 3.36, which is 7% over the accepted value of $\pi \approx 3.14$, which shows that the experiment does not contradict the theory.

The mean value of the number of crossings is $x_{mean} = 5.95$, with a standard deviation of $\sigma = 1.84$, with a mean probability of $p_{mean} = 0.595 \pm 0.184$ for a crossing to occur.

5 Conclusion

An experiment was carried out test the uniformity of the number of crossings of toothpicks in the Buffon's Needle experiment. The the number of crossings was found not to be uniformly distributed. The value of π was estimated with a 7% error, showing the accuracy of the experiment.

References

Kurghinyan, B. (2024, Feb). *Discrete numerical data.*

MSTE. (2015). *Office for mathematics, science, and technology education.* University of Illinois. Retrieved 08 March 2024, from `https://mste.illinois.edu/activity/buffon/`