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ENGS121 Mechanics Lab Section B
Graphical analysis of a spring oscillator

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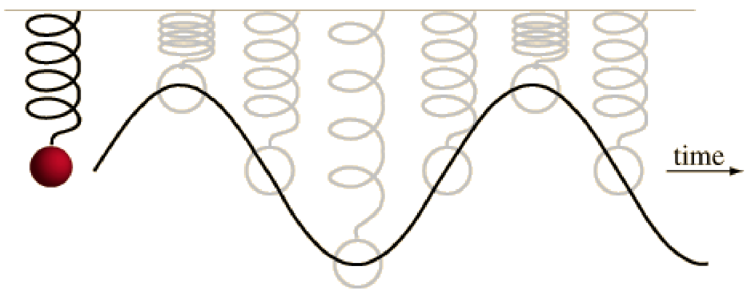
1 Introduction

If you hang a mass to a spring and displace from its equilibrium position, it will start harmonic oscillations.



The simplified formula for the period is:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

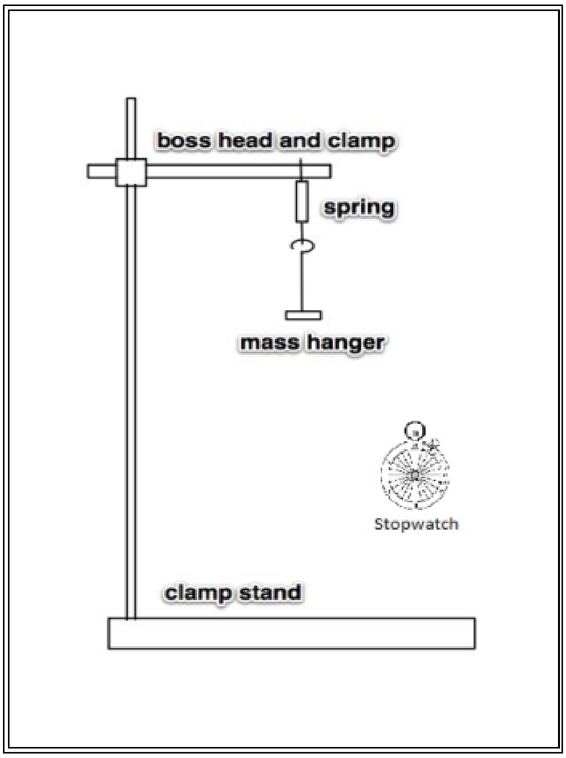
However, the mass of the spring is neglected in this equation. The more accurate equation for the period of a spring oscillator is:

$$T = 2\pi\sqrt{\frac{m + \frac{m_{spring}}{3}}{k}}$$

As we see, the mass of the spring has a $\frac{1}{3}$ coefficient in this equation. An experiment was carried out to test this equation, find the spring constant and this coefficient.

2 Measurements and data

A spring was weighed and mounted on a stand. Different measured weights were hung on the spring. It was displaced from the equilibrium point, and the time for 20 oscillations were measured with a stopwatch, after letting the spring go.



Period was calculated by oscillating the spring a fixed number of times, and measuring the time it took to complete.

Variable	Value	Resolution
Attached weight	Controlled	$1g$
Time of oscillations	Measured	$0.1s$

Table 1: List of variables

Instrument or material	Description
Stand	Lab Stand
Spring	Lab Spring
Weighing scale	With an accuracy of 1g
Weights	Different, to provide sufficient data points
Stopwatch	Smartphone was used

Table 2: List of instruments and materials

Source of error	Type of error	Countermeasures
Oscillations	Random	With lower masses, the spring started to oscillate like a pendulum. The spring was released as vertical as possible, and any experiments with pendulum-like oscillations were disregarded.
Reaction time	Random	Numerous oscillations were done before measuring the time, to lessen the significance of the reaction time.

Table 3: Estimated errors

The weight of the spring was measured 26g, and the time for a constant parameter of 20 oscillations were measured.

m (g)	204	300	399	501	602	703	804	906	1020
t (s)	12.14	14.56	16.24	18.11	19.84	21.29	22.88	24.2	25.6

Table 4: Spring Data

The lower mass of 200 grams were used, because oscillations with any mass less than that causes destabilisation and chaotic pendulum-like oscillations, which would make our calculations less precise. A maximum mass of 1000 grams were used, because with more mass than that the mass would deform the spring and Hooke's law wouldn't be kept. 20 oscillations were used, because for 10, the time for the 200 gram mass was 6 seconds, and reaction time has a greater effect compared to 20 oscillations. And more than 20, would lead to other sources of errors, such as oscillations and loss of energy.

3 Calculations and plots

To use graphical analysis, the equation was linearised by raising both sides of the equation to the power of two. The following equation was obtained:

$$T^2 = \frac{4\pi^2}{k}m + \frac{4\pi^2 \cdot \beta \cdot m_{spring}}{k} \text{ and } T = \frac{t}{N}$$

This equation shows the the dependence of T^2 from m, hence, the periods and squares of the periods were calculated.

m (kg)	0.204	0.300	0.399	0.501	0.602	0.703	0.804	0.906	1.020
T^2 (s^2)	0.3684	0.5300	0.6593	0.8199	0.9841	1.1332	1.3087	1.4641	1.6384

Table 5: Spring Data

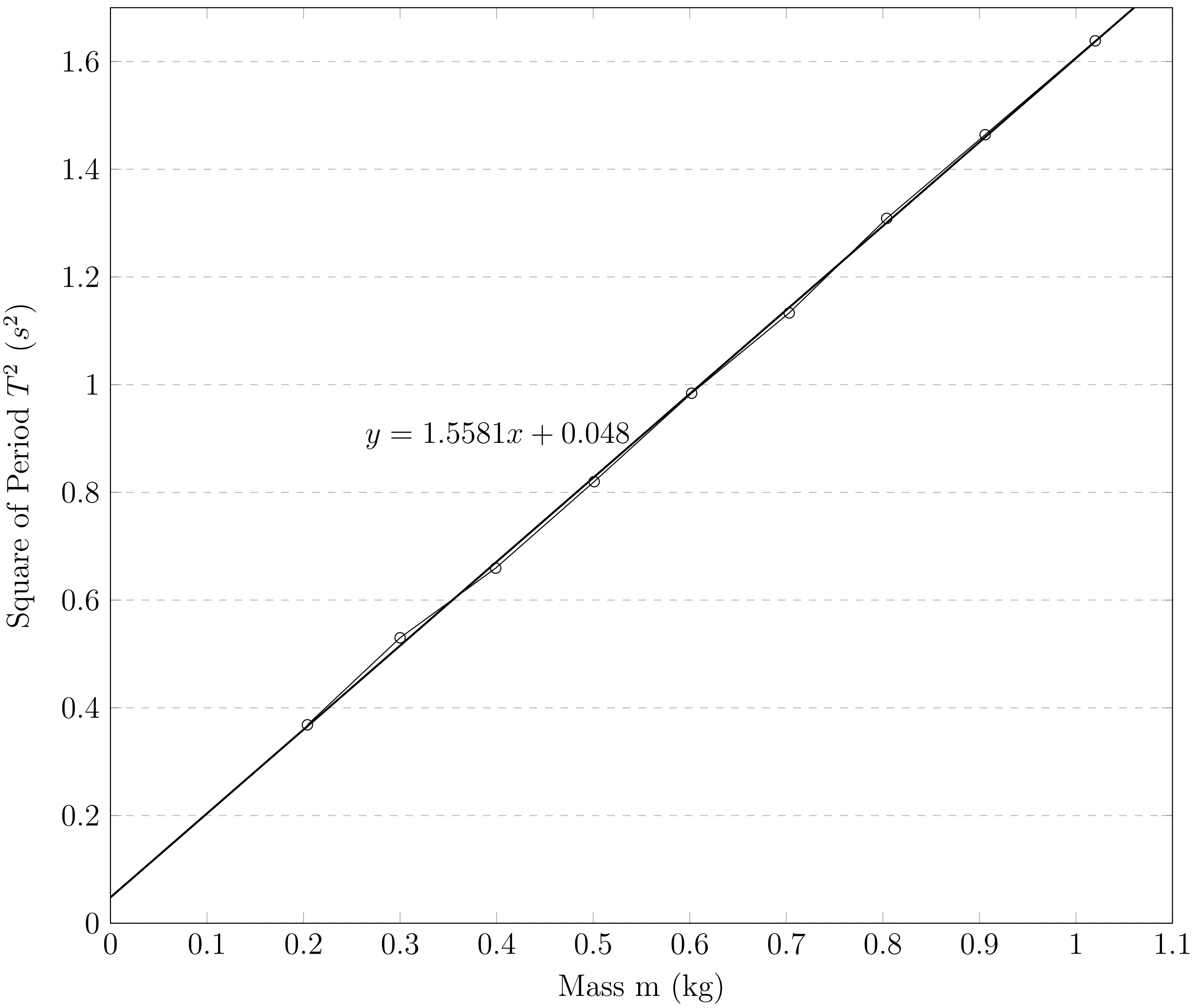


Figure 1: Plot of attached weights and squares of periods

$\mathbf{m^2}$ **(kg)**	0.0416	0.0900	0.1592	0.2510	0.3624	0.4942	0.6464	0.8208	1.0404
$\mathbf{T^2(s^2)}$ **(pred.)**	0.3659	0.5154	0.6697	0.8286	0.9860	1.1433	1.3007	1.4596	1.6373

Table 6: Data for error calculation

Slope and y-intercept error calculation formulas used:

$$(\Delta k)^2 \approx \frac{\sum(y_{predicted} - y_{experiment})^2}{n \cdot (\overline{x^2} - \overline{x}^2) \cdot (n-2)} \text{ and } (\Delta c)^2 \approx \overline{x^2} \cdot (\Delta k)^2$$

4 Evaluation

The slope of the plot is $\frac{4\pi^2}{k} = 1.5581$, which implies that the spring constant is $k = 25.3N/m$. The y-intercept is $\frac{4\pi^2 \cdot \beta \cdot m_{spring}}{k} = 0.048$, hence $\beta = 1.18$. We see that the data points are very close to the trendline, suggesting that the formula is correct. The error for the slope of the plot was calculated 0.012, hence an error of $0.19N/m$ in the spring constant. The y-intercept has an error of 0.0077, which suggest an error of 0.19 in β.

5 Conclusion

An experiment was carried out to test the equation for the period of a spring oscillator. The data confirms the equation, the spring constant was calculated $25.3 \pm 0.2N/m$, however the coefficient of the mass of the spring was found 1.18 ± 0.2, which is far from the theoretical value of $1/3$. The likely cause of this inaccuracy is the relatively heavy masses used in the experiment, compared to the mass of the spring.

References

Khan, S. (2020). *Simple harmonic motion in spring-mass systems review (article).* Khan Academy. Retrieved from `https://www.khanacademy.org/science/oscillations-and-waves-essentials/x9db3ed27fc69f96d:how-are-tall-buildings-protected-from-earthquakes/x9db3ed27fc69f96d:time-period-of-shm/a/simple-harmonic-motion-of-spring-mass-systems-ap`

Kurghinyan, B. (2024, Apr). *Spring pendulum.*