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ENGS124 Electricity and Magnetism Lab Section B

Internal resistance of a voltmeter and variable resistors

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1 Introduction

Part 1: An ideal voltmeter has an infinite resistance across its terminals. However, in real life, no such voltmeters exists. When measuring voltages across a voltage divider, we have the following model:

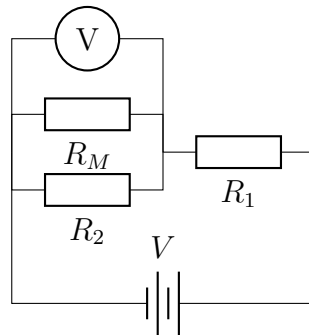


Figure 1: Voltmeter model

Without the resistance of the voltmeter, we have the following formula for the voltage across the second resistor:

$$\frac{V}{R_1 + R_2} = \frac{V_2}{R_2} \implies V_2 = V \frac{R_2}{R_1 + R_2}$$

With high voltmeter resistance, there is no current flowing through it and the measurement is accurate. However, with comparable resistances, we have current flowing the voltmeter and the resistance of the voltmeter is:

$$\frac{V_1}{R_1} = \frac{V_2}{R_2} + \frac{V_2}{R_M} \implies R_M = \frac{V_2 R_1 R_2}{(V - V_2) R_2 - V_2 R_1}$$

Part 2: Variable resistors can be used to provide a reference voltage, depending on the position of a slider/rotor. We have a long strip of wire, two sides are connected to the power supply. We

measure the voltage at a point somewhere along the rheostat.

$$V_1 = V_{\text{supply}} \frac{R_1}{R_1 + R_2} \text{ and } R = \rho \frac{l}{S}$$

Substitute and assume same resistivity and area:

$$\frac{V_1}{V_{\text{supply}}} = \frac{L_1}{L_1 + L_2} \implies L_1 = L \frac{V_1}{V_{\text{supply}}}$$

Part 3: Now, we replace the rheostat with a hand sprayed resistive paper to simulate a one dimensional touchscreen. If we assume the surface is homogenous, we have the following formula for determining the position on the paper:

$$L_1 = L \frac{V_1}{V}$$

2 Measurements and data

Part 1: The circuit in Figure 1 was assembled, different voltages were applied and the voltage across the second resistor was measured with a multimeter in two voltage measurement modes. $R_1 = R_2 = 7.5M\Omega$

V_{supply} (V)	V_{meter} (V)	Mode	R_1 (M Ω)	R_2 (M Ω)
10.00	3.59	20V	7.5	7.5
9.00	3.22	20V	7.5	7.5
8.00	2.87	20V	7.5	7.5
7.00	2.51	20V	7.5	7.5
6.00	2.16	20V	7.5	7.5
5.00	1.79	20V	7.5	7.5
1.802	0.648	2V	7.5	7.5
1.601	0.574	2V	7.5	7.5
1.401	0.503	2V	7.5	7.5
1.200	0.431	2V	7.5	7.5
1.001	0.359	2V	7.5	7.5
0.800	0.286	2V	7.5	7.5

Table 1: Part 1 data

Variable	Value	Resolution
Multimeter mode	Controlled	-
Supply voltage	Controlled	0.01V and 0.001V
Voltages across resistor	Measured	0.01V and 0.001V

Table 2: List of variables

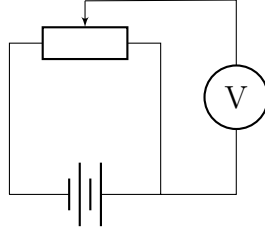
Instrument or material	Description
Voltmeter	A voltmeter
Lab bench power supply	With sufficient voltage range
Resistors	With different resistances
Cables	For the multimeter and power supply

Table 3: List of instruments and materials

Source of error	Type of error	Countermeasures
Power Supply Inaccuracy	Random	Since the voltage meter on the power supply is not very accurate, the voltage was also measured with a multimeter and tuned for improved accuracy.

Table 4: Estimated errors

Part 2: A rheostat was connected to a constant 5.00V power supply, the terminal was varied and the voltage across the left part of the rheostat was measured.



L_1	L_2	V_1 (V)	V (V)
100	0	5.00	5.00
90	10	4.53	5.00
80	20	4.04	5.00
70	30	3.52	5.00
60	40	3.01	5.00
50	50	2.49	5.00
40	60	2.00	5.00
30	70	1.47	5.00
20	80	0.99	5.00
10	90	0.45	5.00
0	100	0.00	5.00

Table 5: Part 2 data

Variable	Value	Resolution
Left part length	Controlled	1 unit
Voltages across left part	Measured	0.01V

Table 6: List of variables

Instrument or material	Description
Voltmeter	A voltmeter
Lab bench power supply	With sufficient voltage range
Variable resistor	a rheostat
Cables	For the multimeter and power supply

Table 7: List of instruments and materials

Source of error	Type of error	Countermeasures
Power Supply Inaccuracy	Random	Since the voltage meter on the power supply is not very accurate, the voltage was also measured with a multimeter and tuned for improved accuracy.
Inconsistent wiper	Systematic	Since the wiper was not securely attached to the rheostat, it had minor movements. To counteract this, all the measurements were taken after pushing the wiper to one direction and letting it go.
Cable Resistance	Systematic	The shortest possible cables were used, to not alter the measurements.
Resistor heating	Systematic	A low voltage was used to limit the heating of the resistor.

Table 8: Estimated errors

Part 3: A resistive paper strip was connected to a constant 5.00V power supply, the terminal was varied and the voltage across the left part of the paper was measured.

L_1 (mm)	L (mm)	V_1 (V)	V (V)
235	235	5.00	5.00
225	235	4.67	5.00
210	235	4.48	5.00
195	235	4.27	5.00
180	235	4.07	5.00
165	235	3.83	5.00
150	235	3.54	5.00
135	235	3.18	5.00
120	235	2.80	5.00
105	235	2.39	5.00
90	235	1.85	5.00
75	235	1.47	5.00
60	235	1.12	5.00
45	235	0.87	5.00
30	235	0.63	5.00
15	235	0.33	5.00
0	235	0.00	5.00

Table 9: Part 3 data

Variable	Value	Resolution
Left part length	Controlled	$1mm$
Voltages across left part	Measured	$0.01V$

Table 10: List of variables

Instrument or material	Description
Voltmeter	A voltmeter
Lab bench power supply	With sufficient voltage range
Resistive paper	Resistive paper
Cables	For the multimeter and power supply

Table 11: List of instruments and materials

Source of error	Type of error	Countermeasures
Power Supply Inaccuracy	Random	Since the voltage meter on the power supply is not very accurate, the voltage was also measured with a multimeter and tuned for improved accuracy.
Cable Resistance	Systematic	The shortest possible cables were used.
Resistor heating	Systematic	A low voltage was used to limit the heating of the resistive paper.
Variable resistance	Random	Since the resistive paper was manually painted, the coating is uneven, thus the resistance is not deterministic from distance and it has some deviation.

Table 12: Estimated errors

3 Calculations and plots

$V_{\text{supply}} (V)$	10.00	9.00	8.00	7.00	6.00	5.00
$V_{\text{meter}} (V)$	3.59	3.22	2.87	2.51	2.16	1.79

Table 13: Part 1 calculations (20V)

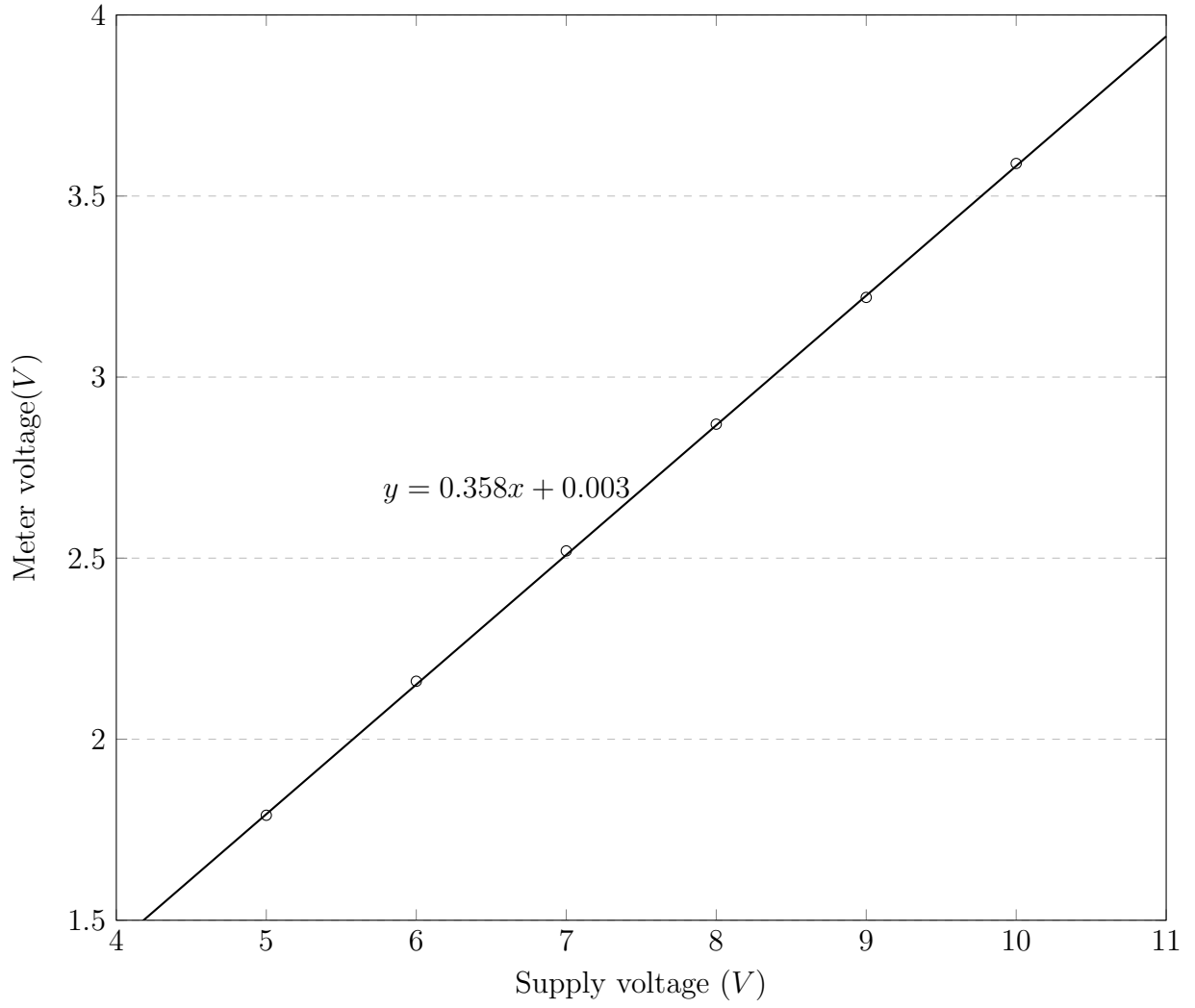


Figure 2: Plot of supply voltage and meter voltage (20V)

$$V_2 = kV + b \implies V = \frac{1}{k}V_2 + c \text{ and } \frac{1}{k} = R_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_M} \right)$$

$$\therefore R_M = \frac{1}{\frac{1}{kR_1} - \frac{1}{R_1} - \frac{1}{R_2}} \approx 9.45 M\Omega$$

V_{supply} (V)	1.802	1.601	1.401	1.200	1.001	0.800
V_{meter} (V)	0.648	0.574	0.503	0.431	0.359	0.286

Table 14: Part 1 calculations (2V)

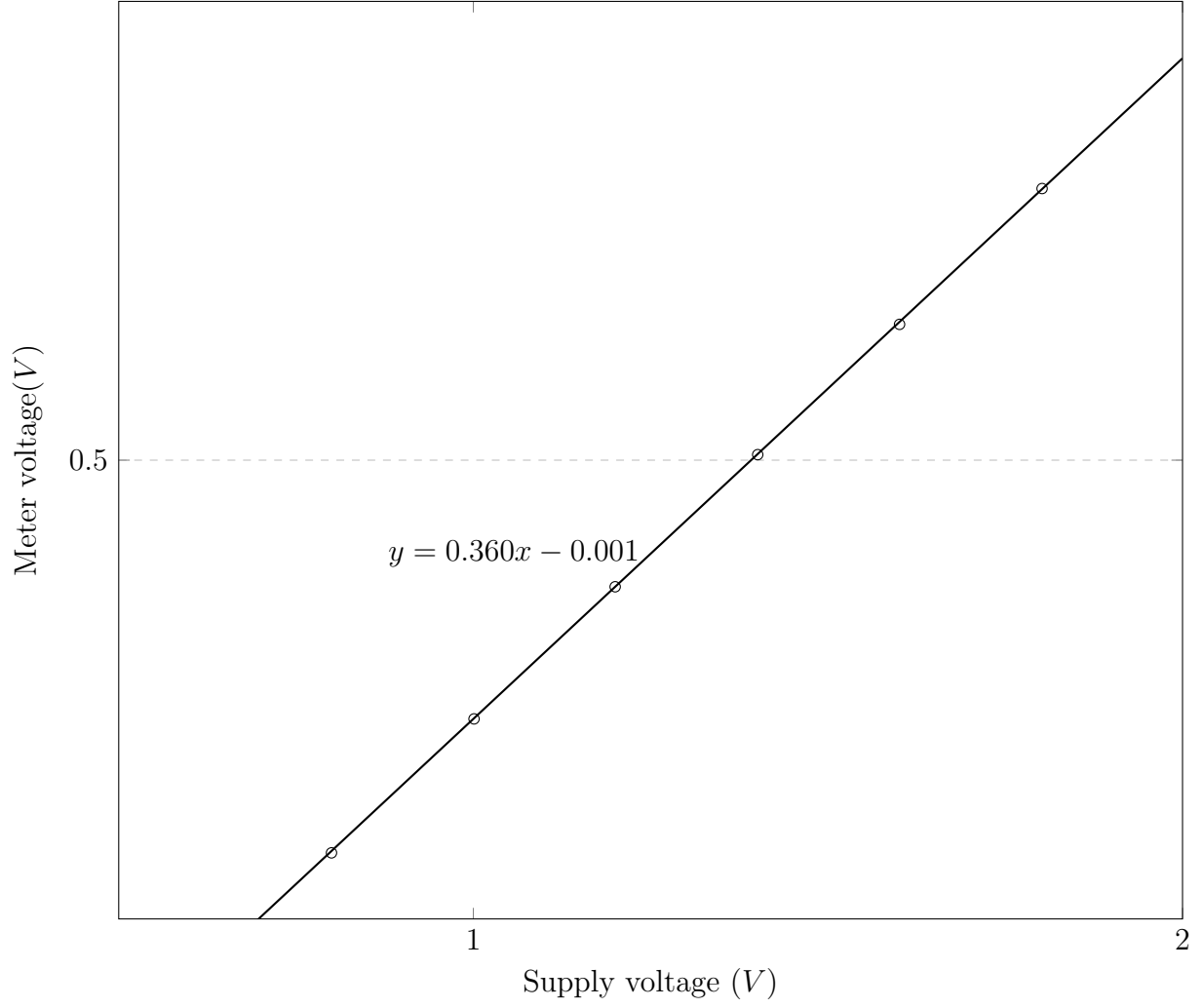


Figure 3: Plot of supply voltage and meter voltage (2V)

$$\therefore R_M = \frac{1}{\frac{1}{kR_1} - \frac{1}{R_1} - \frac{1}{R_2}} \approx 9.64M\Omega$$

$\frac{L_1}{L_1 + L_2}$	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
$\frac{V_1}{V}$	1.000	0.906	0.808	0.704	0.602	0.498	0.400	0.294	0.198	0.09	0.000

Table 15: Part 2 calculations

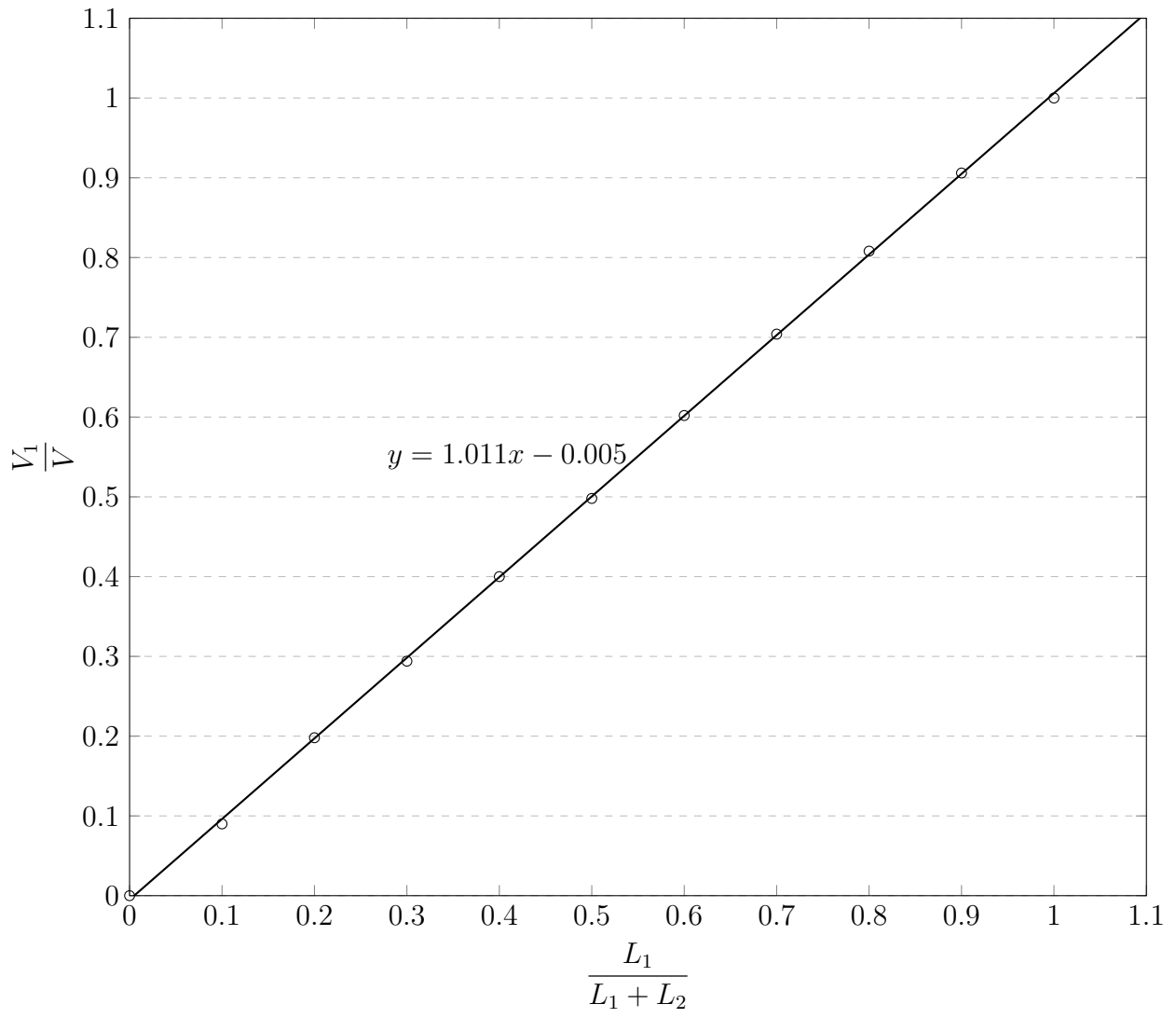


Figure 4: Plot of rheostat position and voltage ratios

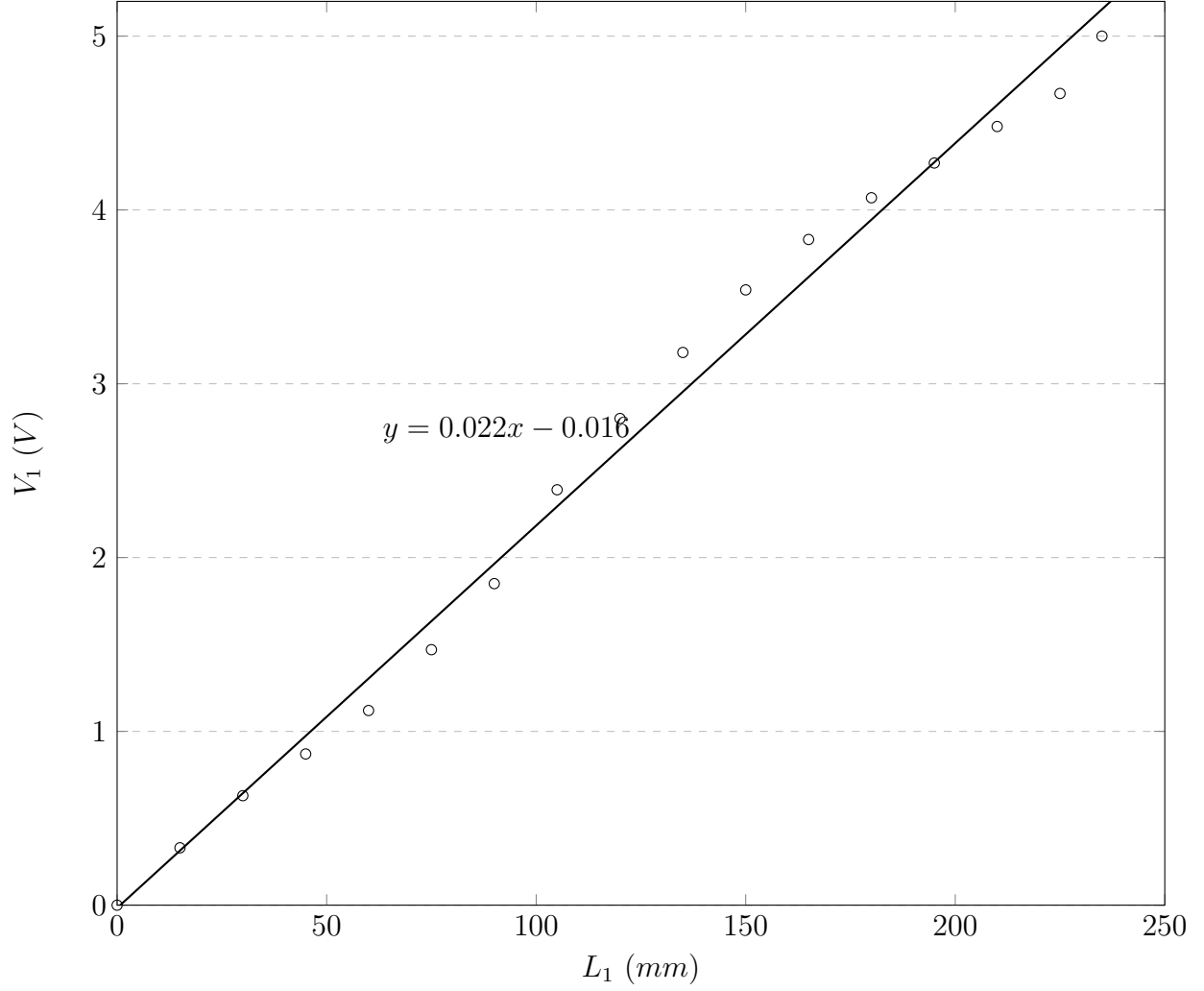


Figure 5: Plot of rheostat position and voltage ratios

4 Evaluation

The internal resistance of the voltmeter in the 20V mode is $9.45M\Omega$, and $9.64M\Omega$ in the 2V mode. For resistors with values $7.5M\Omega$, we have an error of $1 - \frac{3.59}{5.00} \approx 28\%$. Hence, for values in the order of $1.3M\Omega$ we will have an error of approximately 5%. The slope on the second part was found to be close to one.

5 Conclusion

It was found that a voltmeter starts to give wrong values when measuring across resistances in the order of megaohms. We would expect a voltage divider with a 0.5 coefficient, however we get a much lower voltage than expected. Hence, the internal resistance of the voltmeter alters the

measurements. For the third experiment, since the resistive paper is not homogenous, we cannot find a direct relation between the voltage and the position. We need to calibrate beforehand for the best precision.

References

Fluke. (2023, Dec). *What is ohm's law?* Retrieved from <https://www.fluke.com/en-us/learn/blog/electrical/what-is-ohms-law>